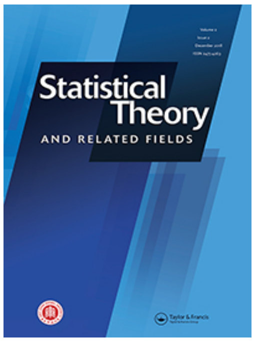

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Dynamic stress–strength reliability estimation of system with survival signature

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ABSTRACT

In this paper, we proposed a dynamic stress–strength model for coherent system. It is supposed that the system consists of n components with initial random strength and each component is subjected to random stresses. The stresses, applied repeatedly at random cycle times, will cause the degradation of strength. In addition, the number of cycles in an interval is assumed to follow a Poisson distribution. In the case of the strength and stress random variables following exponential distributions, the expression for the reliability of the proposed dynamic stress–strength model is derived based on survival signature. The reliability is estimated by using the best linear unbiased estimation (BLUE). Considering the Type-II censored failure times, the best linear unbiased predictors (BLUP) for the unobserved coherent system failure times are developed based on the observed failure times. Monte Carlo simulations are performed to compare the BLUE of parameters with different values and compute the BLUP. A real data set is also analysed for an illustration of the findings.

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Dynamic stress–strength reliability; best linear unbiased estimation; best linear unbiased predictors; survival signature; coherent system

1. Introduction

Stress–strength model is of special importance in engineering applications. A technical system or component may be subjected to randomly occurring environmental stresses such as pressure, temperature, and humidity The survival of the system heavily depends on its resistance. In the simplest form of the stress–strength model, a failure occurs when the strength (or resistance) of the component drops below the stress. In this case the reliability R is defined as the probability that the component's strength is greater than the stress, that is, $R = \Pr[X > Y]$, where X is the random strength of the component and Y is the random stress placed on it. The reliability has been widely studied under various distributional assumptions on X and Y (see, e.g. Johnson 1988; Kotz, Lumelskii, and Pensky 2003) Recently, Liu, Shi, and Bai (2018) studied the estimation for the reliability of a N –M-cold-standby redundancy system based on progressive Type-II censoring sample. Chen and Cheng (2017) discussed estimation of the system stress–strength reliability based on different point estimators and interval estimations when the underlying distribution was exponentiated Pareto distribution. Wang, Geng, and Zhou (2018) developed inferential procedures for the generalised exponential stress–strength model based on generalised pivotal quantity. For more details on the SSR in recent years, see Khan and Jan (2014), Kizilaslan and Nadar (2015), Wang et al. (2018), Rao, Rosaiah, and Babu (2016), Mokhlis, Ibrahim, and Gharieb (2017), Dey, Mazucheli, and Anis (2017), Sales Filho, López Droguett, and Lins (2017), and Bakizi (2014).

In the aforementioned stress–strength model, stress and strength variables are considered to be static. Dynamic (time-dependent) stress–strength modelling may extend more realistic applications in real-life reliability studies than static modelling, which enables us to investigate the dynamic reliability properties of the system. In recent years, much attention has been paid to dynamic stress–strength modelling. Eryilmaz (2013) studied the stress–strength reliability for the case in which the strength of the system is modelled as a stochastic process and the stress is considered to be a usual random variable. Bhuyan and Dewanji (2017a), Bhuyan and Dewanji (2017b), and Bhuyan, Mitra, and Dewanji (2016) presented computation and estimation of reliability as well as identifiability issues under dynamic stress–strength modelling with deterministic strength degradation and cumulative damage due to shocks arriving according to a Poisson process. Cha and Finkelstein (2015) considered a dynamic stress–strength modelling with decreasing strength with time and external shock (stress) process, such as Poisson process. A time-dependent stress–strength model with random strength and constant stresses

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which are repeatedly applied at random cycle times was researched by Siju and Kumar (2016). However, there is little research about dynamic stress–strength modelling for multicomponent system among these literature.

In practice, it is meaningful to predict the unobserved failure times from current available observations for testing system. As mentioned in AL-Hussaini, Abdel-Hamid, and Hashem (2015), to know about plausible warranty limits of products, the experimenters, manufacturers and customers would like to get the bounds of the products life. AL-Hussaini et al. (2015) obtained Bayesian prediction intervals of unobserved progressively Type-II censored data in the presence of competing risks from half-logistic distribution. Basak, Basak, and Balakrishnan (2006) predicted times to failure of censored units in progressively censored samples, including the best linear unbiased predictors (BLUP), the maximum likelihood predictors (MLP), and the conditional median predictors (CMP). Based on the good property of normal distribution, Basak and Balakrishnan (2009) gave the BLUP, modified MLP, approximate MLP and CMP of normal failure times. Balakrishnan, Ng, and Navarro (2011) discussed the BLUP of the Type-II censored (unobserved) system failure times based on the observed failures times. Basak (2014) discussed the problem of predicting failure times for a simple step-stress model under regular and progressive Type-II censoring by computing the MLP and CMP. Asgharzadeh, Valiollahi, and Kundu (2015) considered the classical point predictors and Bayesian predictors of Weibull failure times based on hybrid censored data.

The system signature has been found to be useful for comparisons the performance and quantification of the reliability of coherent system. A lucid review of the theory about signature and its application were presented by Samaniego (2007). For a system with n components and lifetime $T(n)$, the system signature is the vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ with component $p_i = \Pr[T(n) = T_{i:n}]$, $i = 1, 2, \ldots, n$, where $T_{i:n}$ stands for the ith smallest failure time among n components, p_i represents the probability that the system fails upon the failure of the ith component and $\sum_{i=1}^{n} p_i = 1$. Hence, the well-known mixture representation for the survival function (SF) of the systems failure time is

$$\Pr\left[T\left(n\right) > t\right] = \sum_{i=1}^{n} p_i \Pr\left(T_{i:n} > t\right). \qquad (1)$$

Referred to Eryilmaz (2014) and Coolen Coolen-Maturi (2013), Equation (1) can also be expressed as

$$\begin{aligned}\Pr\left[T\left(n\right) > t\right] &= \sum_{k=1}^{n} \frac{\delta_k\left(n\right)}{\binom{n}{k}} \Pr\left[N_n\left(t\right) = k\right] \\ &= \sum_{k=1}^{n} \rho_k \Pr\left[N_n\left(t\right) = k\right], \qquad (2)\end{aligned}$$

where $\delta_k(n)$ denotes the number of path sets of size k for the system, $N_n(t)$ denotes the number of components in the system that function at time t and $\rho_k = \sum_{i=n-k+1}^{n} p_i$. The vector $\boldsymbol{\rho} = (\rho_1, \rho_2, \ldots, \rho_n)$ is called system survival signature. Compared to Samaniego's classical signature, the survival signature has the important advantage that it is applicable to system with multiple types of components. Coolen, Coolen-Maturi, and Al-nefaiee (2013), Aslett, Coolen, and Wilson (2015), and Coolen and Coolen-Maturi (2016) dealt with reliability of systems and networks with multiple types of components based on the survival signature. Pakdaman, Ahmadi, and Doostparast (2017) studied the stress–strength reliability based on survival signature. Except for that, there is little literature on the survival signature for stress–strength reliability of multicomponent systems, especially the dynamic stress–strength reliability (DSSR) of multicomponent systems.

This paper discusses the best linear unbiased estimation (BLUE) for coherent system DSSR with survival signature and the BLUP for unobserved coherent system failure times based on the observed failure times. We study the coherent system with n identical components. Each of these components with initial random strength X is subjected to random stress $Y_i = Y$, $i = 0, 1, 2, \ldots$, which is repeatedly applied at random cycle times to cause the degradation of strength. Let the strength X_i on the ith cycle is given by $X_i = X_{i-1} - c$, $i = 1, 2, \ldots$, where c is a known constant and $X_0 = X$. The aforesaid coherent system has been widely applied in practical engineering fields. For example, a crane is assembled by many identical chains to lift heavier objects repeatedly of various sizes and weights (see Siju & Kumar, 2016).

The random strength of the chain is the bearing capacity and the repeated stresses are the weights of objects. The rest of this paper is organised as follows: In Section 2, the expression of the coherent system DSSR with survival signature is derived. Section 3 presents the exact expressions for the BLUE of the parameters, as well as their variances and covariances. The BLUP of unobserved coherent system failure times of the coherent systems are discussed in Section 4. In Section 5, a simulation study is performed to compare the estimations of parameters with different values, as well as compute the BLUP of the unobserved coherent system failure times by using Monte Carlo simulations and findings are illustrated by tables and figures. Furthermore, a real data set analysis is presented in Section 6. Finally, we conclude the paper in Section 7.

2. DSSR of the coherent system with survival signature

The goal of this section is to derive the expression of the coherent system DSSR with survival signature. In the following two subsections, the expressions of DSSR of the coherent system and its constituent component are obtained, respectively.

2.1. DSSR of the coherent system component

Let X and Y be independent random variables and distributed exponential distributions with different scale parameters $\alpha > 0$ and $\beta > 0$, respectively. The probability density functions (PDF), cumulative distribution functions (CDF) and SFs of X and Y can be expressed as

$$\begin{aligned}
&F_X(x) = 1 - \mathrm{e}^{-\alpha x},\ f_X(x) = \alpha \mathrm{e}^{-\alpha x},\\
&\bar{F}_X(x) = 1 - F_X(x) = \mathrm{e}^{-\alpha x},\ x > 0,\\
&F_Y(y) = 1 - \mathrm{e}^{-\beta y},\ f_Y(y) = \beta \mathrm{e}^{-\beta y},\\
&\bar{F}_Y(y) = 1 - F_Y(y) = \mathrm{e}^{-\beta y},\ y > 0.
\end{aligned}$$

The scale parameters α and β can be readily estimated by the obtained samples of strength X and stress Y. Therefore, it is reasonable to suppose that α and β are known in this paper.

Suppose the stress $Y = y$ is known and let $\omega_i(y),\ i = 0, 1, 2, \ldots,$ be the event that no failure occurs on the ith cycle. The component reliability of success of the first i cycles can be given by

$$\begin{aligned}
r_i^c &= \Pr\left(\omega_1(y) \bigcap \omega_2(y) \bigcap \ldots \bigcap \omega_i(y)\right)\\
&= \Pr\left[(X_1 > y) \bigcap (X_2 > y) \bigcap \ldots \bigcap (X_i > y)\right]\\
&= \Pr\left(X > y + ic\right)\\
&= \mathrm{e}^{-\alpha y}\mathrm{e}^{-i\alpha c}.
\end{aligned} \tag{3}$$

If the stress Y is unknown, the component reliability of success of the first i cycles can be rewritten as

$$R_i^c = \int_0^\infty \mathrm{e}^{-\alpha y}\mathrm{e}^{-i\alpha c} f_Y(y)\,\mathrm{d}y = \frac{\beta}{\alpha+\beta}\mathrm{e}^{-i\alpha c}. \tag{4}$$

Let $N(t)$ denote the number of cycles occurring in the time interval $(0, t]$. Therefore, $N(t)$ is the point process and $\Pr[N(t) = i]$ denotes the probability that exact i cycles occur in time interval $(0, t]$. Suppose that $N(t)$ is the Poisson process, then

$$\Pr[N(t) = i] = \frac{\mathrm{e}^{-(t/\lambda)}(t/\lambda)^i}{i!},\ t \geq 0,\quad i = 0, 1, 2, \ldots, \tag{5}$$

that is, the random variable $N(t)$ follows a Poisson distribution with parameter t/λ. This parameter is equal to the expected number of cycles in time interval $(0, t]$. The Poisson process $\{N(t), t \geq 0\}$ with $N(0) = 0$ has independent increments. Consequently, the component DSSR of the coherent system when $T' = t$ is given by

$$\begin{aligned}
R_{T'}(t) &= \sum_{i=0}^\infty R_i^c \Pr[N(t) = i]\\
&= \sum_{i=0}^\infty \frac{\beta}{\alpha+\beta}\mathrm{e}^{-i\alpha c}\frac{\mathrm{e}^{-(t/\lambda)}(t/\lambda)^i}{i!}\\
&= \frac{\beta}{\alpha+\beta}\mathrm{e}^{-((1-\mathrm{e}^{-\alpha c})t/\lambda)},\quad t \geq 0,
\end{aligned} \tag{6}$$

where the random variable T' implies the lifetime of the component with random initial strength X impacted by the random stress Y, and $R_{T'}(0) = \beta/(\alpha+\beta)$ implies the component static stress–strength reliability at $t = 0$.

2.2. DSSR of the coherent system

Consider the coherent system with n identical components, which are mentioned in Section 2.1, acted upon by the stress $Y_i = Y,\ i = 0, 1, 2, \ldots,$ applied repeatedly at random times. Suppose the stress $Y = y$ is known and the coherent system reliability of success of first i cycles by the use of survival signature $\boldsymbol{\rho} = (\rho_1, \rho_2, \ldots, \rho_n)$ and Equation (3) is derived as

$$\begin{aligned}
r_i &= \sum_{j=1}^n \rho_j \binom{n}{j} (r_i^c)^j (1 - r_i^c)^{n-j}\\
&= \sum_{j=1}^n \rho_j \binom{n}{j} (r_i^c)^j \sum_{k=0}^{n-j} \binom{n-j}{k} (-1)^k (r_i^c)^k\\
&= \sum_{j=1}^n \sum_{k=0}^{n-j} \rho_j \binom{n}{j}\binom{n-j}{k} (-1)^k \left(\mathrm{e}^{-\alpha y}\mathrm{e}^{-i\alpha c}\right)^{j+k},
\end{aligned} \tag{7}$$

where $\rho_j = \delta_j / \binom{n}{j}$ and $\delta_j,\ j = 1, 2, \ldots, n$, denotes the number of path sets of size j for the coherent system. Define $a_j = \sum_{l=1}^j \delta_l \binom{n-l}{j-l} (-1)^{j-l},\ j = 1, 2, \ldots, n$, r_i can be rewritten as

$$r_i = \sum_{j=1}^n a_j \left(\mathrm{e}^{-\alpha y}\mathrm{e}^{-i\alpha c}\right)^j. \tag{8}$$

The vector $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ is called the minimal signature of the system (Balakrishnan et al., 2011). If the stress Y is unknown, the coherent system reliability of success of first i cycles can be rewritten as

$$\begin{aligned}
R_i &= \int_0^\infty \sum_{j=1}^n a_j \left(\mathrm{e}^{-\alpha y}\mathrm{e}^{-i\alpha c}\right)^j f_Y(y)\,\mathrm{d}y\\
&= \sum_{j=1}^n a_j \frac{\beta}{\alpha j + \beta}\mathrm{e}^{-ij\alpha c}.
\end{aligned} \tag{9}$$

Let the random variable T be the lifetime of the coherent system. When $T = t$, from Equation (5), the coherent system DSSR is expressed as

$$\begin{aligned} R_T(t) &= \sum_{i=0}^{\infty} R_i \Pr[N(t) = i] \\ &= \sum_{i=0}^{\infty}\sum_{j=1}^{n} a_j \frac{\beta}{\alpha j + \beta} \mathrm{e}^{-ij\alpha c} \frac{\mathrm{e}^{-(t/\lambda)}(t/\lambda)^i}{i!} \\ &= \sum_{j=1}^{n} a_j \frac{\beta}{\alpha j + \beta} \mathrm{e}^{-(t/\lambda)(1-\mathrm{e}^{-j\alpha c})}, \quad t \ge 0. \end{aligned} \tag{10}$$

Define $b_j = \beta/(\alpha j + \beta)$ and $\sigma_j = 1 - \mathrm{e}^{-j\alpha c}$. DSSR of the coherent system can be rewritten as

$$R_T(t) = \sum_{j=1}^{n} a_j b_j \mathrm{e}^{-((\sigma_j t)/\lambda)}, \quad t \ge 0. \tag{11}$$

Consequently, the CDF and PDF of T can be given by

$$F_T(t) = 1 - R_T(t) = 1 - \sum_{j=1}^{n} a_j b_j \mathrm{e}^{-((\sigma_j t)/\lambda)}, \quad t \ge 0, \tag{12}$$

$$f_T(t) = \frac{\mathrm{d}F_T(t)}{\mathrm{d}t} = \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-((\sigma_j t)/\lambda)}, \quad t \ge 0. \tag{13}$$

When $t = 0$, $R_T(0) = \sum_{j=1}^{n} a_j b_j = \sum_{j=1}^{n}(a_j\beta/(\alpha j + \beta))$ implies the static stress–strength reliability of coherent system.

3. Best linear unbiased estimation

Suppose m independent coherent systems with the same survival signature $\boldsymbol{\rho}$ are placed on a life test. The test is terminated when the rth (where $r \le m$ is prefixed) system fails. Additionally, the Type-II censored samples $\{T_{1:m}, T_{2:m}, \ldots, T_{r:m}\}$ of T are observed.

Denote the moments, the product moments, and the variance–covariance matrix of $\mathbf{T_r} = (T_{1:m}, T_{2:m}, \ldots, T_{r:m})'$ by $\boldsymbol{\mu}_r = (\mu_{1:m}, \mu_{2:m}, \ldots, \mu_{r:m})'$, $(\mu_{1,1:m}, \mu_{1,2:m}, \ldots, \mu_{1,r:m}, \mu_{2,1:m}, \mu_{2,2:m}, \ldots, \mu_{2,r:m}, \ldots, \mu_{r,1:m}, \mu_{r,2:m}, \ldots, \mu_{r,r:m})$ and $\Sigma_\mathbf{r} = (\sigma_{i,j:m})_{r\times r}$, where $i, j = 1, 2, \ldots, r$, $\mu_{i:m} = E(T_{i:m})$, $\mu_{i,j:m} = \mu_{j,i:m} = E(T_{i:m}T_{j:m})$ and $\sigma_{i,j:m} = \sigma_{j,i:m} = \mu_{i,j:m} - \mu_{i:m}\mu_{j:m}$.

Define $T^* = T/\lambda$, $T^*_{i:m} = T_{i:m}/\lambda$, $i = 1, 2, \ldots, r$ and $\mathbf{T}^*_\mathbf{r} = (T^*_{1:m}, T^*_{2:m}, \ldots T^*_{r:m})'$. The CDF and PDF of T^* can be expressed as

$$F_{T^*}(t) = 1 - \sum_{j=1}^{n} a_j b_j \mathrm{e}^{-\sigma_j t}, \quad t \ge 0, \tag{14}$$

$$f_{T^*}(t) = \frac{\mathrm{d}F_{T^*}(t)}{\mathrm{d}t} = \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t}, \quad t \ge 0. \tag{15}$$

Denote the moments, the product moments, and the variance–covariance matrix of $\mathbf{T}^*_\mathbf{r} = (T^*_{1:m}, T^*_{2:m}, \ldots T^*_{r:m})'$ by $\boldsymbol{\mu}^*_r = (\mu^*_{1:m}, \mu^*_{2:m}, \ldots, \mu^*_{r:m})$, $(\mu^*_{1,1:m}, \ldots, \mu^*_{1,r:m}, \mu^*_{2,1:m}, \ldots, \mu^*_{2,r:m}, \ldots, \mu^*_{r,1:m}, \ldots, \mu^*_{r,r:m})$ and $\Sigma^*_r = (\sigma^*_{i,j:m})_{r\times r}$, where $i, j = 1, 2, \ldots, r$, $\mu^*_{i:m} = E(T^*_{i:m})$, $\mu^*_{i,j:m} = \mu^*_{j,i:m} = E(T^*_{i:m}T^*_{j:m})$ and $\sigma^*_{i,j:m} = \sigma^*_{j,i:m} = \mu^*_{i,j:m} - \mu^*_{i:m}\mu^*_{j:m}$. As a result, the moments and the variance–covariance matrix can be expressed as

$$E(\mathbf{T_r}) = E\left[\lambda \mathbf{T}^*_\mathbf{r}\right] = \boldsymbol{\mu}_\mathbf{r} = \lambda \boldsymbol{\mu}^*_\mathbf{r}, \tag{16}$$

$$Cov(\mathbf{T_r}) = Cov\left[\lambda \mathbf{T}^*_\mathbf{r}\right] = \Sigma_\mathbf{r} = \lambda^2 \Sigma^*_\mathbf{r}. \tag{17}$$

Define the generalised variance as

$$\begin{aligned} Q &= (\mathbf{T_r} - \boldsymbol{\mu}_\mathbf{r})' \Sigma^{*-1}_\mathbf{r} (\mathbf{T_r} - \boldsymbol{\mu}_\mathbf{r}) \\ &= \left(\mathbf{T_r} - \lambda\boldsymbol{\mu}^*_\mathbf{r}\right)' \Sigma^{*-1}_\mathbf{r} \left(\mathbf{T_r} - \lambda\boldsymbol{\mu}^*_\mathbf{r}\right) \\ &= \mathbf{T}'_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \mathbf{T_r} - 2\lambda \boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \mathbf{T_r} + \lambda^2 \boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \boldsymbol{\mu}^*_\mathbf{r}. \end{aligned} \tag{18}$$

According to Balakrishnan et al. (2011), by minimising Q with respect to λ, the BLUE $\hat{\lambda}$ of λ and its variance can be obtained as

$$\hat{\lambda} = \left(\frac{\boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r}}{\boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \boldsymbol{\mu}^*_\mathbf{r}}\right) \mathbf{T_r} = \mathbf{g_r} \mathbf{T_r} = \sum_{i=1}^{r} g_{r,i:m} T_{i:m}, \tag{19}$$

$$Var\left(\hat{\lambda}\right) = \frac{\lambda^2}{\boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \boldsymbol{\mu}^*_\mathbf{r}}, \tag{20}$$

where

$$\left(\frac{\boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r}}{\boldsymbol{\mu}^{*\prime}_\mathbf{r} \Sigma^{*-1}_\mathbf{r} \boldsymbol{\mu}^*_\mathbf{r}}\right) = \mathbf{g_r} = (g_{r,1:m}, g_{r,2:m}, \ldots, g_{r,r:m}),$$
$$r = 1, 2, \ldots, m$$

is the coefficient vector of the censored samples $\{T_{1:m}, T_{2:m}, \ldots, T_{r:m}\}$. To compute $\hat{\lambda}$, the expressions of the moments $\mu^*_{1:m}, \mu^*_{2:m}, \ldots, \mu^*_{r:m}$ and variance–covariance matrix Σ^* will be given in the following two subsections.

3.1. Moments $\mu^*_{1:m}, \mu^*_{2:m}, \ldots, \mu^*_{r:m}$ of $T^*_{1:m}, T^*_{2:m}, \ldots T^*_{r:m}$

On the basis of Equations (14)–(15), the PDF of $T^*_{1:m}$ is derived as

$$\begin{aligned} f_{T^*_{1:m}}(t) &= m[1 - F_{T^*}(t)]^{m-1} f_{T^*}(t) \\ &= m\left\{\sum_{j=1}^{n} a_j b_j \mathrm{e}^{-\sigma_j t}\right\}^{m-1} \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t} \\ &= \left\{\begin{array}{l} m \displaystyle\sum_{\substack{j_1,\ldots,j_n \ge 0 \\ j_1+\ldots+j_n = m-1}} \binom{m-1}{j_1, j_2, \ldots, j_n} \\ \left\{a_1 b_1 \mathrm{e}^{-\sigma_1 t}\right\}^{j_1} \left\{a_2 b_2 \mathrm{e}^{-\sigma_2 t}\right\}^{j_2} \\ \times \ldots \times \left\{a_n b_n \mathrm{e}^{-\sigma_n t}\right\}^{j_n} \displaystyle\sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t} \end{array}\right. \end{aligned}$$

$$= \begin{cases} m \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-1}} \binom{m-1}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)t}. \end{cases} \tag{21}$$

As a consequence, the expectation of $T^*_{1:m}$ can be given by

$$\begin{aligned} \mu^*_{1:m} &= E\left(T^*_{1:m}\right) \\ &= \begin{cases} \int_0^\infty tm \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-1}} \binom{m-1}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)t} \mathrm{d}t \end{cases} \\ &= \begin{cases} m \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-1}} \binom{m-1}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} \frac{a_j b_j \sigma_j}{(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)^2}. \end{cases} \end{aligned} \tag{22}$$

In addition, the values of $\mu^*_{2:m} = E(T^*_{2:m}), \mu^*_{3:m} = E(T^*_{3:m}), \dots, \mu^*_{r:m} = E(T^*_{r:m})$ can be obtained by the use of triangle rule (see Theorem 5.3.1 in Arnold, Balakrishnan, & Nagaraja, 2008),

$$sE\left(T^*_{s+1:m}\right) + (m-s)\,E\left(T^*_{s:m}\right) = mE\left(T^*_{s:m-1}\right), \tag{23}$$

where $s = 1,2,\dots,r-1$.

Based on Equation (23), it is readily to obtain that

$$\mu^*_{2:m} = m\mu^*_{1:m-1} - (m-1)\,\mu^*_{1:m},$$

where

$$\begin{aligned} \mu^*_{1:m-1} &= E\left(T^*_{1:m-1}\right) \\ &= \begin{cases} (m-1) \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-2}} \binom{m-2}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} \frac{a_j b_j \sigma_j}{(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)^2}. \end{cases} \end{aligned}$$

Similarly, the values of $\mu^*_{3:m} = E(T^*_{3:m}), \dots, \mu^*_{r:m}$ $E(T^*_{r:m})$ can be derived.

3.2. Variance–covariance matrix $\Sigma^*_{\mathbf{r}}$ of $T^*_{1:m}, T^*_{2:m}, \dots T^*_{r:m}$

In order to obtain $\Sigma^*_{\mathbf{r}}$, we firstly compute the product moments $\mu^*_{i,j:m}$, $i,j = 1,2,\dots,r$. Since the symmetry of the variance–matrix $\Sigma^*_{\mathbf{r}}$, we just need to compute $\mu^*_{i,j:m}$, $i \leq j$.

Case I: $i = j$, $i = 1,2,\dots,r$.

Based on Equation (21), the second moment of $T^*_{1:m}$ can be obtained by

$$\begin{aligned} \mu^*_{1,1:m} &= E\left(T^{*2}_{1:m}\right) \\ &= \begin{cases} \int_0^\infty t^2 m \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-1}} \binom{m-1}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)t} \mathrm{d}t \end{cases} \\ &= \begin{cases} m \sum_{\substack{j_1,\dots,j_n\geq 0\\ j_1+\dots+j_n=m-1}} \binom{m-1}{j_1,j_2,\dots,j_n} \\ a_1^{j_1} a_2^{j_2} \dots a_n^{j_n} b_1^{j_1} b_2^{j_2} \dots b_n^{j_n} \\ \times \sum_{j=1}^{n} \frac{2a_j b_j \sigma_j}{(\sigma_1 j_1+\sigma_2 j_2+\dots+\sigma_n j_n+\sigma_j)^3}. \end{cases} \end{aligned} \tag{24}$$

Also, the values of $\mu^*_{2,2:m} = E(T^{*2}_{2:m}), \mu^*_{3,3:m} = E(T^{*2}_{3:m}), \dots, \mu^*_{r,r:m} = E(T^{*2}_{r:m})$ can be obtained by the use of triangle rule $sE(T^{*2}_{s+1:m}) + (m-s)E(T^{*2}_{s:m}) = mE(T^{*2}_{s:m-1})$, where $s = 1,2,\dots,r-1$.

Case II: $i \neq j$, $j = 2,3,\dots,r$ and $i = 1,2,\dots,j-1$.

Consider the joint density of $(T^*_{s:m}, T^*_{s+1:m})$, for $1 \leq s \leq r-1$, given by

$$f_{T^*_{s:m},T^*_{s+1:m}}(t,u) = \begin{cases} \frac{m!}{(s-1)!\,(m-s-1)!}[F_{T^*}(t)]^{s-1} \\ \times[1-F_{T^*}(u)]^{m-s-1} f_{T^*}(t) \\ f_{T^*}(u), \quad 0 \leq t < u. \end{cases} \tag{25}$$

In the light of Equations (14)–(15), the joint density function of $(T^*_{s:m}, T^*_{s+1:m})$ can be rewritten as

$$\begin{aligned} &f_{T^*_{s:m},T^*_{s+1:m}}(t,u) \\ &= \begin{cases} \frac{m!}{(s-1)!\,(m-s-1)!}\left(1-\sum_{j=1}^{n} a_j b_j \mathrm{e}^{-\sigma_j t}\right)^{s-1} \\ \left(\sum_{j=1}^{n} a_j b_j \mathrm{e}^{-\sigma_j u}\right)^{m-s-1} \\ \times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t} \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j u} \end{cases} \end{aligned}$$

$$
= \left\{ \begin{array}{l}
\frac{m!}{(s-1)!\,(m-s-1)!} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \\
\left(\sum_{j=1}^{n} a_j b_j \mathrm{e}^{-\sigma_j t} \right)^k \\
\times \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \binom{m-s-1}{j_1, j_2, \ldots, j_n} \left(a_1 b_1 \mathrm{e}^{-\sigma_1 u}\right)^{j_1} \\
\left(a_2 b_2 \mathrm{e}^{-\sigma_2 u}\right)^{j_2} \ldots \left(a_n b_n \mathrm{e}^{-\sigma_n u}\right)^{j_n} \\
\times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t} \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j u}
\end{array} \right.
$$

$$
= \left\{ \begin{array}{l}
\frac{m!}{(s-1)!\,(m-s-1)!} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \\
\sum_{\substack{k_1,\ldots,k_n \geq 0 \\ k_1+\ldots+k_n=k}} \binom{k}{k_1, k_2, \ldots, k_n} \left(a_1 b_1 \mathrm{e}^{-\sigma_1 t}\right)^{k_1} \\
\times \left(a_2 b_2 \mathrm{e}^{-\sigma_2 t}\right)^{k_2} \ldots \left(a_n b_n \mathrm{e}^{-\sigma_n t}\right)^{k_n} \\
\times \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j t} \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \\
\binom{m-s-1}{j_1, j_2, \ldots, j_n} \\
\times \left(a_1 b_1 \mathrm{e}^{-\sigma_1 u}\right)^{j_1} \left(a_2 b_2 \mathrm{e}^{-\sigma_2 u}\right)^{j_2} \ldots \left(a_n b_n \mathrm{e}^{-\sigma_n u}\right)^{j_n} \\
\sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\sigma_j u}
\end{array} \right.
$$

$$
= \left\{ \begin{array}{l}
\frac{m!}{(s-1)!\,(m-s-1)!} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \\
\sum_{\substack{k_1,\ldots,k_n \geq 0 \\ k_1+\ldots+k_n=k}} \binom{k}{k_1, k_2, \ldots, k_n} a_1^{k_1} a_2^{k_2} \ldots a_n^{k_n} \\
\times b_1^{k_1} b_2^{k_2} \ldots b_n^{k_n} \sum_{j=1}^{n} a_j b_j \sigma_j \\
\mathrm{e}^{-(\sigma_1 k_1 + \sigma_2 k_2 + \ldots + \sigma_n k_n + \sigma_j) t} \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \\
\binom{m-s-1}{j_1, j_2, \ldots, j_n} \\
\times a_1^{j_1} a_2^{j_2} \ldots a_n^{j_n} b_1^{j_1} b_2^{j_2} \ldots b_n^{j_n} \sum_{j=1}^{n} a_j b_j \sigma_j \\
\mathrm{e}^{-(\sigma_1 j_1 + \sigma_2 j_2 + \ldots + \sigma_n j_n + \sigma_j) u}, \ 0 \leq t < u < \infty.
\end{array} \right.
$$

Define $\eta_j = \sigma_1 k_1 + \sigma_2 k_2 + \cdots + \sigma_n k_n + \sigma_j$ and $\varphi_l = \sigma_1 j_1 + \sigma_2 j_2 + \cdots + \sigma_n j_n + \sigma_l$, $l = 1, 2, \ldots, n$, then

$$
f_{T^*_{s:m}, T^*_{s+1:m}}(t, u) = \left\{ \begin{array}{l}
\frac{m!}{(s-1)!\,(m-s-1)!} \\
\sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \sum_{\substack{k_1,\ldots,k_n \geq 0 \\ k_1+\ldots+k_n=k}} \\
\binom{k}{k_1, k_2, \ldots, k_n} a_1^{k_1} a_2^{k_2} \ldots a_n^{k_n} \\
\times b_1^{k_1} b_2^{k_2} \ldots b_n^{k_n} \sum_{j=1}^{n} a_j b_j \sigma_j \mathrm{e}^{-\eta_j t} \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \\
\binom{m-s-1}{j_1, j_2, \ldots, j_n} a_1^{j_1} a_2^{j_2} \ldots a_n^{j_n} \\
\times b_1^{j_1} b_2^{j_2} \ldots b_n^{j_n} \sum_{l=1}^{n} a_l b_l \sigma_l \mathrm{e}^{-\varphi_l u}, \ 0 \leq t < u < \infty.
\end{array} \right. \tag{26}
$$

Moreover, define $\varepsilon_j(u) = \int_0^u t\mathrm{e}^{-\eta_j t}\mathrm{d}t = 1/\eta_j^2(1 - \eta_j u\mathrm{e}^{-\eta_j u} - \mathrm{e}^{-\eta_j u})$. Therefore, the product moment of $(T^*_{s:m}, T^*_{s+1:m})$ can be expressed as

$$
\mu^*_{s,s+1:m} = E\left(T^*_{s:m}, T^*_{s+1:m}\right) = \int_0^\infty \int_0^u t u f_{T^*_{s:m}, T^*_{s+1:m}}(t, u)\, \mathrm{d}t\mathrm{d}u
$$

$$
= \left\{ \begin{array}{l}
\int_0^\infty \frac{m!u}{(s-1)!\,(m-s-1)!} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \\
\sum_{\substack{k_1,\ldots,k_n \geq 0 \\ k_1+\ldots+k_n=k}} \binom{k}{k_1, k_2, \ldots, k_n} \\
a_1^{k_1} a_2^{k_2} \ldots a_n^{k_n} b_1^{k_1} b_2^{k_2} \ldots b_n^{k_n} \\
\times \sum_{j=1}^{n} a_j b_j \sigma_j \varepsilon_j(u) \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \\
\binom{m-s-1}{j_1, j_2, \ldots, j_n} a_1^{j_1} a_2^{j_2} \ldots a_n^{j_n} b_1^{j_1} b_2^{j_2} \ldots b_n^{j_n} \\
\sum_{l=1}^{n} a_l b_l \sigma_l \mathrm{e}^{-\varphi_l u} \mathrm{d}u
\end{array} \right.
$$

$$
= \left\{ \begin{array}{l}
\frac{m!}{(s-1)!\,(m-s-1)!} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \\
\sum_{\substack{k_1,\ldots,k_n \geq 0 \\ k_1+\ldots+k_n=k}} \binom{k}{k_1, k_2, \ldots, k_n} \\
a_1^{k_1} a_2^{k_2} \ldots a_n^{k_n} b_1^{k_1} b_2^{k_2} \ldots b_n^{k_n} \\
\times \sum_{\substack{j_1,\ldots,j_n \geq 0 \\ j_1+\ldots+j_n=m-s-1}} \binom{m-s-1}{j_1, j_2, \ldots, j_n} \\
a_1^{k_1+j_1} a_2^{k_2+j_2} \ldots a_n^{k_n+j_n} b_1^{k_1+j_1} b_2^{k_2+j_2} \ldots b_n^{k_n+j_n} \\
\sum_{j=1}^{n} \sum_{l=1}^{n} a_j a_l b_j b_l \sigma_j \sigma_l \\
\times \int_0^\infty u\mathrm{e}^{-\varphi_l u} \varepsilon_j(u)\, \mathrm{d}u
\end{array} \right.
$$

$$
= \begin{cases} \dfrac{m!}{(s-1)!\,(m-s-1)!}\displaystyle\sum_{k=0}^{s-1}(-1)^k\binom{s-1}{k} \\ \displaystyle\sum_{\substack{k_1,\ldots,k_n\geq 0\\ k_1+\ldots+k_n=k}}\binom{k}{k_1,k_2,\ldots,k_n} \\ \displaystyle\sum_{\substack{j_1,\ldots,j_n\geq 0\\ j_1+\ldots+j_n=m-s-1}}\binom{m-s-1}{j_1,j_2,\ldots,j_n} \\ \times a_1^{k_1+j_1}a_2^{k_2+j_2}\ldots a_n^{k_n+j_n}b_1^{k_1+j_1}b_2^{k_2+j_2}\ldots b_n^{k_n+j_n} \\ \displaystyle\sum_{j=1}^{n}\sum_{l=1}^{n}a_ja_lb_jb_l\sigma_j\sigma_l\frac{1}{\eta_j^2} \\ \times\left[\dfrac{1}{\varphi_l^2}-\dfrac{1}{(\eta_j+\varphi_l)^2}-\dfrac{2\eta_j}{(\eta_j+\varphi_l)^3}\right]. \end{cases} \tag{27}
$$

Based on Equation (27), the values of $\mu^*_{1,2:m}, \mu^*_{2,3:m}, \ldots, \mu^*_{r-1,r:m}$ can be readily obtained. And then, all other product moments can be computed by the use of rectangle rule (see Theorem 5.3.9 in Arnold et al., 2008)

$$
\begin{aligned}
&(b-1)\,E\left(T^*_{b:m}T^*_{d:m}\right)+(d-b)\,E\left(T^*_{b-1:m}T^*_{d:m}\right)\\
&\quad+(m-d+1)\,E\left(T^*_{b-1:m}T^*_{d-1:m}\right)\\
&\quad=mE\left(T^*_{b:m-1}T^*_{d:m-1}\right),
\end{aligned} \tag{28}
$$

where $2\leq b<d\leq r$. Hence, according to $\sigma^*_{i,j:m}=\sigma^*_{j,i:m}=\mu^*_{i,j:m}-\mu^*_{i:m}\mu^*_{j:m}$, the variance–covariance matrix $\Sigma^*_{\mathbf{r}}$ can be readily calculated.

4. Best linear unbiased predictors

According to Balakrishnan et al. (2011), the BLUP of the Type-II censored (unobserved) coherent system failure times based on the observed failure times are discussed in this section.

From Equation (19), the BLUE of the parameter λ based on the first r observed failure times can be rewritten as

$$
\hat{\lambda}_r=\hat{\lambda}=\sum_{i=1}^{r}g_{r,i:m}T_{i:m}. \tag{29}
$$

Then, from the results of Doganaksoy and Balakrishnan (1997), by equating $\hat{\lambda}_r=\hat{\lambda}_{r+1}$, the BLUP of $T_{r+1:m}$ can be obtained as

$$
\hat{T}_{r+1:m}=\sum_{i=1}^{r}\left(\frac{g_{r,i:m}-g_{r+1,i:m}}{g_{r+1,r+1:m}}\right)T_{i:m}, \tag{30}
$$

where

$$
\begin{aligned}
\mathbf{g}_{\mathbf{r+1}}&=\left(\frac{\boldsymbol{\mu}^{*\prime}_{\mathbf{r+1}}\Sigma^{*-1}_{\mathbf{r+1}}}{\boldsymbol{\mu}^{*\prime}_{\mathbf{r+1}}\Sigma^{*-1}_{\mathbf{r+1}}\boldsymbol{\mu}^{*}_{\mathbf{r+1}}}\right)\\
&=\left(g_{r+1,1:m},g_{r+1,2:m},\ldots,g_{r+1,r+1:m}\right).
\end{aligned}
$$

The predictor of $T_{k:m}$ $(r+1<k\leq m)$ can be obtained in a similar manner by regarding the BLUP of $T_{r+1:m}, T_{r+2:m},\ldots,T_{k-1:m}$ as observed failure times in the BLUE. Finally, the BLUP of $T_{k:m}$ $(r<k\leq m)$ and its variance based on the first r failure times can be computed as

$$
\hat{T}_{k:m}=\sum_{i=1}^{r}h_iT_{i:m}, \tag{31}
$$

$$
Var\left(\hat{T}_{k:m}\right)=\lambda^2\mathbf{h}\Sigma^*_{\mathbf{r}}\mathbf{h}', \tag{32}
$$

where $\mathbf{h}=(h_1,h_2,\ldots,h_r)$ can be calculated by equating $\hat{\lambda}_r=\hat{\lambda}_{r+1}=\cdots=\hat{\lambda}_k$.

5. Numerical simulation

To study the performance of the BLUE $\hat{\lambda}$ of λ and BLUP of the Type-II censored (unobserved) coherent system failure times based on the observed failures times mentioned in the preceding sections, an extensive Monte Carlo simulation study is carried out in this section.

Six different coherent systems with different system signatures and structure, listed in Table 1, are considered. It is supposed that these systems are made up of the same components, which are mentioned in Section 2 with known parameters which are fixed by $\alpha=1$, $\beta=9$ and $c=0.1$. The coefficients and $\mathrm{Var}(\hat{\lambda})/\lambda^2$ for these different coherent systems with different censored schemes are computed and the results are presented in Tables 2–7.

Set the parameter $\lambda=0.5, 2, 8$ and 15, respectively. For each value of the parameter, the Type-II censored samples of different censored schemes are generated. The BLUE $\hat{\lambda}$ and standard error $\mathrm{SE}=\sqrt{\mathrm{Var}(\hat{\lambda})}$ are computed based on the coefficients in Tables 2–7. Repeat this process 10,000 times. The results are presented in Tables 8–11.

Additionally, for each coherent system, we generate the complete samples of the system by fixing $\lambda=0.5$ and $m=13$. Let $r=7$. The predictors and corresponding variances $V=\mathrm{Var}(T_{i:m})$ $(i=8,9,\ldots,13)$

Table 1. The description of six different coherent systems.

System	System structure	Survival signature	Minimal signature
System 1	3 components 2-out-of-3 system	$\rho=(0,2/3,1)$	$\mathbf{a}=(0,3,-2)$
System 2	3 components Parallel system	$\rho=(1,1,1)$	$\mathbf{a}=(3,-3,1)$
System 3	3 components Parallel-Series system	$\rho=(1/3,1,1)$	$\mathbf{a}=(1,1,-1)$
System 4	4 components 3-out-of-4 system	$\rho=(0,0,1,1)$	$\mathbf{a}=(0,0,2,-1)$
System 5	4 components compound system	$\rho=(0,1/2,3/4,1)$	$\mathbf{a}=(0,3,-3,1)$
System 6	4 components compound system	$\rho=(1/4,1/2,3/4,1)$	$\mathbf{a}=(1,0,1,-1)$

Table 2. Coefficients of $\hat{\lambda}$ and corresponding $\mathrm{Var}(\hat{\lambda})/\lambda^2$ for System 1.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$\mathrm{Var}(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0159	−0.0071	0.0025	0.0074	0.0092	0.0098	0.0103	0.0105	0.0107	0.0108	0.0109	0.0109	0.0106	0.0549
(13, 12)	−0.0171	−0.0076	0.0027	0.0079	0.0099	0.0105	0.0110	0.0113	0.0115	0.0116	0.0117	0.0228		0.0589
(13, 11)	−0.0186	−0.0083	0.0029	0.0087	0.0106	0.0115	0.0120	0.0123	0.0125	0.0126	0.0367			0.0640
(13, 10)	−0.0204	−0.0091	0.0033	0.0094	0.0118	0.0126	0.0132	0.0135	0.0137	0.0531				0.0704
(13, 9)	−0.0228	−0.0102	0.0036	0.0107	0.0130	0.0141	0.0146	0.0150	0.0726					0.0786
(13, 8)	−0.0259	−0.0116	0.0040	0.0120	0.0148	0.0159	0.0166	0.0968						0.0894
(13, 7)	−0.0301	−0.0136	0.0047	0.0139	0.0172	0.0185	0.1276							0.1042
(13, 6)	−0.0361	−0.0164	0.0055	0.0166	0.0205	0.1690								0.1254
(13, 5)	−0.0454	−0.0208	0.0067	0.0208	0.2283									0.1587
(13, 4)	−0.0615	−0.0287	0.0086	0.3203										0.2169
(13, 3)	−0.0951	−0.0457	0.4745											0.3396
(13, 2)	−0.1913	0.7780												0.6787
(10, 10)	−0.0148	−0.0020	0.0080	0.0119	0.0131	0.0137	0.0140	0.0142	0.0142	0.0139				0.0716
(10, 9)	−0.0163	−0.0022	0.0088	0.0130	0.0144	0.0150	0.0154	0.0156	0.0304					0.0788
(10, 8)	−0.0183	−0.0025	0.0099	0.0146	0.0161	0.0168	0.0172	0.0504						0.0885
(10, 7)	−0.0211	−0.0029	0.0113	0.0167	0.0185	0.0192	0.0754							0.1017
(10, 6)	−0.0250	−0.0035	0.0133	0.0197	0.0218	0.1083								0.1206
(10, 5)	−0.0308	−0.0045	0.0163	0.0243	0.1542									0.1495
(10, 4)	−0.0408	−0.0063	0.0213	0.2242										0.1986
(10, 3)	−0.0607	−0.0102	0.3413											0.2981
(10, 2)	−0.1149	0.5649												0.5647
(8, 8)	−0.0126	0.0035	0.0132	0.0163	0.0172	0.0177	0.0178	0.0175						0.0898
(8, 7)	−0.0143	0.0040	0.0150	0.0184	0.0195	0.0199	0.0392							0.1017
(8, 6)	−0.0167	0.0046	0.0175	0.0214	0.0226	0.0671								0.1189
(8, 5)	−0.0204	0.0056	0.0212	0.0259	0.1052									0.1450
(8, 4)	−0.0265	0.0070	0.0272	0.1621										0.1887
(8, 3)	−0.0385	0.0094	0.2567											0.2746
(8, 2)	−0.0698	0.4352												0.4982
(5, 5)	−0.0029	0.0193	0.0269	0.0285	0.0283									0.1447
(5, 4)	−0.0036	0.0239	0.0333	0.0686										0.1802
(5, 3)	−0.0052	0.0323	0.1330											0.2479
(5, 2)	−0.0095	0.2530												0.4141

Table 3. Coefficients of $\hat{\lambda}$ and corresponding $\mathrm{Var}(\hat{\lambda})/\lambda^2$ for System 2.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$\mathrm{Var}(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0016	0.0033	0.0044	0.0047	0.0048	0.0049	0.0048	0.0048	0.0047	0.0046	0.0044	0.0042	0.0039	0.0377
(13, 12)	−0.0017	0.0035	0.0047	0.0048	0.0052	0.0050	0.0051	0.0051	0.0049	0.0048	0.0046	0.0084		0.0395
(13, 11)	−0.0018	0.0037	0.0048	0.0054	0.0050	0.0057	0.0051	0.0054	0.0052	0.0051	0.0136			0.0417
(13, 10)	−0.0019	0.0040	0.0051	0.0056	0.0057	0.0057	0.0057	0.0057	0.0055	0.0195				0.0446
(13, 9)	−0.0020	0.0041	0.0058	0.0059	0.0061	0.0062	0.0061	0.0061	0.0263					0.0481
(13, 8)	−0.0023	0.0047	0.0060	0.0067	0.0066	0.0068	0.0066	0.0343						0.0526
(13, 7)	−0.0026	0.0052	0.0068	0.0072	0.0074	0.0075	0.0438							0.0585
(13, 6)	−0.0028	0.0057	0.0077	0.0082	0.0083	0.0557								0.0666
(13, 5)	−0.0035	0.0069	0.0089	0.0096	0.0710									0.0783
(13, 4)	−0.0043	0.0082	0.0109	0.0920										0.0965
(13, 3)	−0.0059	0.0107	0.1240											0.1290
(13, 2)	−0.0098	0.1800												0.2022
(10, 10)	−0.0002	0.0051	0.0061	0.0063	0.0063	0.0063	0.0061	0.0059	0.0056	0.0051				0.0491
(10, 9)	−0.0002	0.0055	0.0065	0.0067	0.0067	0.0067	0.0065	0.0063	0.0113					0.0523
(10, 8)	−0.0002	0.0059	0.0070	0.0072	0.0073	0.0072	0.0070	0.0186						0.0565
(10, 7)	−0.0002	0.0065	0.0077	0.0079	0.0080	0.0079	0.0273							0.0622
(10, 6)	−0.0003	0.0073	0.0086	0.0089	0.0089	0.0381								0.0701
(10, 5)	−0.0004	0.0084	0.0099	0.0103	0.0519									0.0814
(10, 4)	−0.0005	0.0101	0.0119	0.0707										0.0990
(10, 3)	−0.0009	0.0130	0.0989											0.1301
(10, 2)	−0.0019	0.1480												0.1990
(8, 8)	0.0015	0.0070	0.0078	0.0079	0.0078	0.0076	0.0072	0.0065						0.0614
(8, 7)	0.0016	0.0076	0.0085	0.0086	0.0085	0.0082	0.0146							0.0667
(8, 6)	0.0018	0.0085	0.0094	0.0095	0.0094	0.0246								0.0742
(8, 5)	0.0020	0.0097	0.0108	0.0109	0.0374									0.0852
(8, 4)	0.0024	0.0115	0.0128	0.0547										0.1024
(8, 3)	0.0029	0.0146	0.0803											0.1325
(8, 2)	0.0037	0.1246												0.1986
(5, 5)	0.0068	0.0124	0.0126	0.0120	0.0106									0.0984
(5, 4)	0.0078	0.0144	0.0145	0.0255										0.1141
(5, 3)	0.0096	0.0177	0.0473											0.1425
(5, 2)	0.0132	0.0842												0.2042

for the Type-II censored (unobserved) coherent systems failure times based on the observed failures times are computed by using the first r samples. The results are presented in Table 12 and the comparison of the predictors and the last $m-r$ samples, as well as the variation of V are shown in Figure 1.

From Tables 8–11, for each coherent system, it is readily seen that SE decreases as r increases, when m

Table 4. Coefficients of $\hat{\lambda}$ and corresponding $\mathrm{Var}(\hat{\lambda})/\lambda^2$ for System 3.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$\mathrm{Var}(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0130	−0.0021	0.0056	0.0083	0.0089	0.0090	0.0089	0.0086	0.0082	0.0077	0.0070	0.0063	0.0058	0.0636
(13, 12)	−0.0137	−0.0023	0.0060	0.0088	0.0094	0.0096	0.0094	0.0091	0.0087	0.0081	0.0074	0.0129		0.0674
(13, 11)	−0.0146	−0.0024	0.0064	0.0093	0.0101	0.0102	0.0101	0.0097	0.0093	0.0086	0.0218			0.0718
(13, 10)	−0.0157	−0.0026	0.0068	0.0100	0.0108	0.0109	0.0108	0.0105	0.0100	0.0328				0.0772
(13, 9)	−0.0171	−0.0028	0.0074	0.0109	0.0118	0.0119	0.0118	0.0114	0.0465					0.0842
(13, 8)	−0.0190	−0.0032	0.0082	0.0121	0.0130	0.0132	0.0130	0.0636						0.0934
(13, 7)	−0.0215	−0.0036	0.0093	0.0137	0.0147	0.0149	0.0855							0.1060
(13, 6)	−0.0252	−0.0043	0.0108	0.0159	0.0171	0.1147								0.1239
(13, 5)	−0.0306	−0.0054	0.0130	0.0192	0.1557									0.1513
(13, 4)	−0.0398	−0.0073	0.0166	0.2182										0.1977
(13, 3)	−0.0581	−0.0114	0.3226											0.2907
(13, 2)	−0.1067	0.5196												0.5364
(10, 10)	−0.0108	0.0029	0.0099	0.0116	0.0117	0.0113	0.0107	0.0097	0.0085	0.0077				0.0830
(10, 9)	−0.0116	0.0031	0.0107	0.0125	0.0126	0.0122	0.0115	0.0105	0.0176					0.0895
(10, 8)	−0.0127	0.0034	0.0117	0.0137	0.0138	0.0134	0.0125	0.0308						0.0979
(10, 7)	−0.0142	0.0038	0.0130	0.0152	0.0154	0.0149	0.0483							0.1092
(10, 6)	−0.0163	0.0043	0.0149	0.0174	0.0175	0.0716								0.1253
(10, 5)	−0.0194	0.0050	0.0177	0.0206	0.1042									0.1497
(10, 4)	−0.0247	0.0061	0.0222	0.1531										0.1905
(10, 3)	−0.0349	0.0080	0.2345											0.2698
(10, 2)	−0.0611	0.3869												0.4725
(8, 8)	−0.0077	0.0079	0.0139	0.0146	0.0140	0.0128	0.0111	0.0097						0.1040
(8, 7)	−0.0085	0.0087	0.0153	0.0161	0.0155	0.0141	0.0232							0.1147
(8, 6)	−0.0096	0.0099	0.0173	0.0182	0.0174	0.0422								0.1295
(8, 5)	−0.0112	0.0115	0.0202	0.0212	0.0692									0.1519
(8, 4)	−0.0140	0.0141	0.0248	0.1097										0.1890
(8, 3)	−0.0194	0.0188	0.1767											0.2598
(8, 2)	−0.0329	0.3021												0.4360
(5, 5)	0.0034	0.0208	0.0227	0.0198	0.0162									0.1673
(5, 4)	0.0040	0.0247	0.0269	0.0430										0.1990
(5, 3)	0.0050	0.0315	0.0891											0.2573
(5, 2)	0.0069	0.1768												0.3966

Table 5. Coefficients of $\hat{\lambda}$ and corresponding $Var(\hat{\lambda})/\lambda^2$ for System 4.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$\mathrm{Var}(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0256	−0.0312	−0.0299	−0.0191	−0.0030	0.0104	0.0177	0.0207	0.0219	0.0228	0.0235	0.0242	0.0248	0.0941
(13, 12)	−0.0286	−0.0350	−0.0335	−0.0214	−0.0034	0.0117	0.0199	0.0232	0.0246	0.0255	0.0263	0.0543		0.1055
(13, 11)	−0.0326	−0.0398	−0.0382	−0.0244	−0.0039	0.0133	0.0226	0.0264	0.0280	0.0290	0.0908			0.1203
(13, 10)	−0.0379	−0.0463	−0.0444	−0.0284	−0.0046	0.0154	0.0263	0.0307	0.0325	0.1375				0.1399
(13, 9)	−0.0451	−0.0552	−0.0529	−0.0339	−0.0055	0.0184	0.0313	0.0365	0.2004					0.1669
(13, 8)	−0.0556	−0.0681	−0.0654	−0.0419	−0.0068	0.0226	0.0386	0.2892						0.2064
(13, 7)	−0.0722	−0.0885	−0.0851	−0.0547	−0.0090	0.0294	0.4215							0.2686
(13, 6)	−0.1016	−0.1247	−0.1200	−0.0772	−0.0128	0.6276								0.3768
(13, 5)	−0.1618	−0.1990	−0.1918	−0.1236	0.9685									0.5874
(13, 4)	−0.3124	−0.3848	−0.3715	1.6099										1.0497
(13, 3)	−0.8040	−0.9921	3.1374											2.2389
(13, 2)	−3.1897	8.3557												6.2630
(10, 10)	−0.0328	−0.0331	−0.0210	−0.0007	0.0165	0.0252	0.0285	0.0301	0.0313	0.0324				0.1233
(10, 9)	−0.0382	−0.0386	−0.0245	−0.0008	0.0192	0.0294	0.0332	0.0350	0.0733					0.1438
(10, 8)	−0.0459	−0.0463	−0.0294	−0.0010	0.0230	0.0352	0.0398	0.1283						0.1727
(10, 7)	−0.0572	−0.0578	−0.0368	−0.0013	0.0287	0.0439	0.2070							0.2158
(10, 6)	−0.0757	−0.0765	−0.0488	−0.0019	0.0379	0.3278								0.2861
(10, 5)	−0.1100	−0.1115	−0.0712	−0.0029	0.5247									0.4150
(10, 4)	−0.1870	−0.1899	−0.1216	0.8737										0.6878
(10, 3)	−0.4103	−0.4175	1.6212											1.3692
(10, 2)	−1.3593	3.9045												3.5562
(8, 8)	−0.0367	−0.0284	−0.0057	0.0182	0.0316	0.0366	0.0389	0.0407						0.1553
(8, 7)	−0.0447	−0.0347	−0.0070	0.0221	0.0384	0.0445	0.0956							0.1893
(8, 6)	−0.0572	−0.0444	−0.0090	0.0282	0.0491	0.1766								0.2426
(8, 5)	−0.0790	−0.0614	−0.0126	0.0389	0.3068									0.3353
(8, 4)	−0.1240	−0.0966	−0.0201	0.5335										0.5228
(8, 3)	−0.2433	−0.1901	0.9883											0.9791
(8, 2)	−0.7034	2.2546												2.4031
(5, 5)	−0.0320	0.0039	0.0412	0.0590	0.0654									0.2526
(5, 4)	−0.0452	0.0054	0.0580	0.1724										0.3568
(5, 3)	−0.0752	0.0087	0.3746											0.5906
(5, 2)	−0.1727	0.8556												1.2870

is fixed. SE decreases as m increases, when r is fixed. The deviation between the BLUE $\hat{\lambda}$ and fixed value of parameter λ reaches minimum when $r = [m/2]$, where $f(x) = [x]$ is the top integral function. Additionally, the larger the fixed value of parameter λ, the greater the variance.

Table 6. Coefficients of $\hat{\lambda}$ and corresponding $Var(\hat{\lambda})/\lambda^2$ for System 5.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$Var(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0170	−0.0149	−0.0076	0.0005	0.0055	0.0081	0.0094	0.0104	0.0113	0.0121	0.0127	0.0132	0.0133	0.0682
(13, 12)	−0.0186	−0.0164	−0.0083	0.0005	0.0060	0.0090	0.0103	0.0114	0.0125	0.0132	0.0140	0.0286		0.0750
(13, 11)	−0.0209	−0.0180	−0.0099	0.0010	0.0067	0.0097	0.0118	0.0126	0.0139	0.0147	0.0464			0.0838
(13, 10)	−0.0235	−0.0213	−0.0097	−0.0005	0.0084	0.0111	0.0131	0.0144	0.0157	0.0677				0.0953
(13, 9)	−0.0274	−0.0239	−0.0132	0.0019	0.0082	0.0132	0.0152	0.0166	0.0942					0.1107
(13, 8)	−0.0327	−0.0284	−0.0147	0.0003	0.0112	0.0151	0.0182	0.1280						0.1318
(13, 7)	−0.0397	−0.0357	−0.0178	0.0007	0.0131	0.0188	0.1742							0.1622
(13, 6)	−0.0513	−0.0452	−0.0237	0.0011	0.0168	0.2407								0.2092
(13, 5)	−0.0705	−0.0628	−0.0325	0.0012	0.3439									0.2890
(13, 4)	−0.1089	−0.0972	−0.0507	0.5155										0.4443
(13, 3)	−0.2046	−0.1832	0.8406											0.8035
(13, 2)	−0.5436	1.6695												1.8620
(10, 10)	−0.0181	−0.0117	−0.0011	0.0070	0.0111	0.0132	0.0148	0.0160	0.0170	0.0174				0.0892
(10, 9)	−0.0206	−0.0133	−0.0012	0.0080	0.0126	0.0150	0.0168	0.0182	0.0382					0.1013
(10, 8)	−0.0240	−0.0155	−0.0015	0.0093	0.0147	0.0175	0.0195	0.0639						0.1184
(10, 7)	−0.0289	−0.0187	−0.0018	0.0111	0.0176	0.0210	0.0973							0.1429
(10, 6)	−0.0363	−0.0236	−0.0024	0.0139	0.0221	0.1437								0.1800
(10, 5)	−0.0483	−0.0316	−0.0033	0.0184	0.2132									0.2408
(10, 4)	−0.0709	−0.0465	−0.0051	0.3270										0.3541
(10, 3)	−0.1230	−0.0812	0.5350											0.6067
(10, 2)	−0.2921	1.0237												1.3329
(8, 8)	−0.0177	−0.0069	0.0057	0.0132	0.0169	0.0192	0.0210	0.0218						0.1121
(8, 7)	−0.0208	−0.0082	0.0067	0.0156	0.0199	0.0226	0.0491							0.1323
(8, 6)	−0.0257	−0.0101	0.0082	0.0191	0.0244	0.0854								0.1633
(8, 5)	−0.0335	−0.0133	0.0105	0.0248	0.1378									0.2134
(8, 4)	−0.0476	−0.0190	0.0147	0.2214										0.3042
(8, 3)	−0.0784	−0.0317	0.3712											0.5003
(8, 2)	−0.1724	0.7044												1.0518
(5, 5)	−0.0113	0.0097	0.0245	0.0316	0.0350									0.1817
(5, 4)	−0.0151	0.0128	0.0325	0.0856										0.2432
(5, 3)	−0.0232	0.0192	0.1708											0.3715
(5, 2)	−0.0453	0.3452												0.7153

Table 7. Coefficients of $\hat{\lambda}$ and corresponding $\mathrm{Var}(\hat{\lambda})/\lambda^2$ for System 6.

(m,r)	$g_{r,1:m}$	$g_{r,2:m}$	$g_{r,3:m}$	$g_{r,4:m}$	$g_{r,5:m}$	$g_{r,6:m}$	$g_{r,7:m}$	$g_{r,8:m}$	$g_{r,9:m}$	$g_{r,10:m}$	$g_{r,11:m}$	$g_{r,12:m}$	$g_{r,13:m}$	$\mathrm{Var}(\hat{\lambda})/\lambda^2$
(13, 13)	−0.0175	−0.0050	0.0059	0.0103	0.0110	0.0108	0.0101	0.0093	0.0084	0.0075	0.0069	0.0066	0.0068	0.0742
(13, 12)	−0.0187	−0.0054	0.0064	0.0110	0.0119	0.0116	0.0109	0.0099	0.0090	0.0081	0.0074	0.0145		0.0797
(13, 11)	−0.0201	−0.0058	0.0069	0.0118	0.0127	0.0124	0.0117	0.0107	0.0096	0.0087	0.0237			0.0856
(13, 10)	−0.0218	−0.0063	0.0074	0.0128	0.0138	0.0134	0.0126	0.0116	0.0104	0.0353				0.0925
(13, 9)	−0.0237	−0.0068	0.0081	0.0139	0.0150	0.0146	0.0138	0.0126	0.0503					0.1009
(13, 8)	−0.0263	−0.0076	0.0089	0.0154	0.0166	0.0162	0.0152	0.0699						0.1117
(13, 7)	−0.0297	−0.0086	0.0100	0.0174	0.0187	0.0183	0.0960							0.1263
(13, 6)	−0.0345	−0.0101	0.0116	0.0202	0.0218	0.1322								0.1474
(13, 5)	−0.0421	−0.0125	0.0140	0.0245	0.1847									0.1801
(13, 4)	−0.0550	−0.0167	0.0179	0.2668										0.2371
(13, 3)	−0.0817	−0.0258	0.4058											0.3554
(13, 2)	−0.1565	0.6766												0.6782
(10, 10)	−0.0149	0.0018	0.0119	0.0141	0.0136	0.0123	0.0108	0.0095	0.0087	0.0089				0.0968
(10, 9)	−0.0163	0.0020	0.0131	0.0155	0.0149	0.0135	0.0119	0.0104	0.0195					0.1062
(10, 8)	−0.0180	0.0022	0.0144	0.0171	0.0164	0.0149	0.0131	0.0333						0.1170
(10, 7)	−0.0201	0.0024	0.0161	0.0191	0.0184	0.0166	0.0523							0.1308
(10, 6)	−0.0230	0.0027	0.0184	0.0218	0.0210	0.0792								0.1499
(10, 5)	−0.0274	0.0032	0.0218	0.0259	0.1188									0.1786
(10, 4)	−0.0347	0.0038	0.0273	0.1807										0.2272
(10, 3)	−0.0494	0.0046	0.2861											0.3246
(10, 2)	−0.0887	0.4880												0.5820
(8, 8)	−0.0109	0.0086	0.0167	0.0169	0.0149	0.0128	0.0113	0.0111						0.1214
(8, 7)	−0.0122	0.0097	0.0188	0.0190	0.0168	0.0144	0.0253							0.1363
(8, 6)	−0.0139	0.0110	0.0214	0.0216	0.0191	0.0457								0.1551
(8, 5)	−0.0163	0.0129	0.0250	0.0252	0.0764									0.1820
(8, 4)	−0.0203	0.0158	0.0308	0.1254										0.2257
(8, 3)	−0.0280	0.0211	0.2098											0.3107
(8, 2)	−0.0480	0.3719												0.5289
(5, 5)	0.0035	0.0242	0.0244	0.0199	0.0180									0.1953
(5, 4)	0.0042	0.0293	0.0297	0.0466										0.2373
(5, 3)	0.0053	0.0379	0.0995											0.3081
(5, 2)	0.0075	0.2071												0.4747

From Table 12 and Figure 1, for each coherent system, it is observed that the predictors are closed to the real value of samples and V increases as i increases.

6. Real data analysis

In this section, a real data set in Lawless (2011) is analysed by using the proposed methods. The data set

Table 8. BLUE and standard errors for 6 different coherent systems ($\lambda = 0.5$).

	System 1		System 2		System 3		System 4		System 5		System 6	
(m, r)	$\hat{\lambda}$	*SE*	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE
(13, 13)	0.5762	0.1119	0.5281	0.1025	0.5425	0.1368	0.5623	0.1725	0.5576	0.1456	0.5471	0.1491
(13, 12)	0.5671	0.1127	0.5127	0.1019	0.5413	0.1405	0.5613	0.1823	0.5483	0.1501	0.5475	0.1545
(13, 11)	0.5494	0.1123	0.5036	0.1029	0.5351	0.1434	0.5625	0.1951	0.5407	0.1565	0.5467	0.1600
(13, 10)	0.5508	0.1163	0.4801	0.1013	0.5329	0.1481	0.5669	0.2120	0.5341	0.1649	0.5453	0.1658
(13, 9)	0.5264	0.1154	0.4663	0.1023	0.5201	0.1509	0.5730	0.2341	0.5257	0.1749	0.5422	0.1722
(13, 8)	0.5090	0.1167	0.4349	0.0997	0.5123	0.1566	0.5877	0.2670	0.5226	0.1897	0.5382	0.1799
(13, 7)	0.5025	0.1216	0.4312	0.1043	0.5012	0.1631	0.6055	0.3138	0.5240	0.2110	0.5331	0.1895
(13, 6)	0.4874	0.1258	0.3891	0.1004	0.4906	0.1727	0.6448	0.3958	0.5283	0.2416	0.5283	0.2028
(13, 5)	0.4804	0.1344	0.3689	0.1032	0.4804	0.1869	0.7246	0.5553	0.5409	0.2908	0.5229	0.2219
(13, 4)	0.4761	0.1479	0.3349	0.1041	0.4684	0.2082	0.8968	0.9188	0.5736	0.3823	0.5170	0.2517
(13, 3)	0.4450	0.1598	0.3014	0.1082	0.4595	0.2477	1.3024	1.9487	0.6668	0.5977	0.5134	0.3061
(13, 2)	0.4906	0.2206	0.2599	0.1169	0.4564	0.3343	2.5234	6.3151	0.9010	1.2295	0.5397	0.4445
(10, 10)	0.5735	0.1270	0.5279	0.1169	0.5509	0.1587	0.5691	0.1998	0.5630	0.1681	0.5511	0.1715
(10, 9)	0.5614	0.1283	0.5019	0.1147	0.5405	0.1617	0.5709	0.2165	0.5516	0.1756	0.5513	0.1796
(10, 8)	0.5364	0.1275	0.4849	0.1153	0.5352	0.1674	0.5788	0.2405	0.5452	0.1876	0.5488	0.1877
(10, 7)	0.5399	0.1347	0.4731	0.1180	0.5280	0.1744	0.5900	0.2741	0.5399	0.2041	0.5453	0.1973
(10, 6)	0.5227	0.1384	0.4351	0.1152	0.5168	0.1829	0.6137	0.3283	0.5357	0.2273	0.5400	0.2091
(10, 5)	0.5000	0.1426	0.4043	0.1153	0.5020	0.1943	0.6616	0.4262	0.5361	0.2631	0.5347	0.2260
(10, 4)	0.5007	0.1576	0.3751	0.1180	0.4836	0.2111	0.7573	0.6281	0.5554	0.3305	0.5284	0.2519
(10, 3)	0.4819	0.1738	0.3303	0.1191	0.4695	0.2439	0.9911	1.1597	0.6056	0.4717	0.5226	0.2977
(10, 2)	0.5040	0.2248	0.2898	0.1293	0.4676	0.3214	1.6348	3.0830	0.7473	0.8628	0.5405	0.4123
(8, 8)	0.5744	0.1423	0.5396	0.1337	0.5463	0.1762	0.5807	0.2288	0.5636	0.1887	0.5517	0.1922
(8, 7)	0.5574	0.1439	0.5045	0.1303	0.5404	0.1830	0.5842	0.2541	0.5508	0.2003	0.5504	0.2032
(8, 6)	0.5338	0.1454	0.4738	0.1290	0.5314	0.1912	0.5985	0.2948	0.5418	0.2189	0.5467	0.2153
(8, 5)	0.5237	0.1528	0.4540	0.1325	0.5176	0.2017	0.6253	0.3621	0.5368	0.2480	0.5418	0.2311
(8, 4)	0.4921	0.1574	0.3937	0.1260	0.5011	0.2178	0.6851	0.4954	0.5422	0.2991	0.5365	0.2549
(8, 3)	0.4992	0.1817	0.3729	0.1358	0.4768	0.2430	0.8310	0.8223	0.5670	0.4011	0.5298	0.2953
(8, 2)	0.4682	0.2086	0.3179	0.1417	0.4657	0.3075	1.2215	1.8935	0.6714	0.6885	0.5326	0.3873
(5, 5)	0.5769	0.1810	0.5325	0.1670	0.5468	0.2237	0.5905	0.2968	0.5717	0.2437	0.5505	0.2433
(5, 4)	0.5445	0.1839	0.4900	0.1655	0.5354	0.2388	0.6098	0.3642	0.5590	0.2757	0.5490	0.2675
(5, 3)	0.5140	0.1941	0.4378	0.1653	0.5180	0.2627	0.6660	0.5118	0.5614	0.3421	0.5428	0.3013
(5, 2)	0.4794	0.2166	0.3750	0.1695	0.4803	0.3025	0.8187	0.9288	0.5891	0.4982	0.5322	0.3667

Table 9. BLUE and standard errors for 6 different coherent systems ($\lambda = 2$).

	System 1		System 2		System 3		System 4		System 5		System 6	
(m, r)	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE
(13, 13)	2.3013	0.4469	2.1317	0.4139	2.1836	0.5508	2.2568	0.6922	2.2312	0.5826	2.1922	0.5973
(13, 12)	2.2485	0.4469	2.0596	0.4093	2.1582	0.5602	2.2584	0.7335	2.1923	0.6002	2.1958	0.6198
(13, 11)	2.2199	0.4536	2.0057	0.4098	2.1538	0.5770	2.2618	0.7844	2.1619	0.6258	2.1941	0.6420
(13, 10)	2.1390	0.4515	1.9338	0.4082	2.1307	0.5921	2.2767	0.8514	2.1344	0.6590	2.1910	0.6663
(13, 9)	2.0930	0.4589	1.8601	0.4079	2.0989	0.6090	2.3065	0.9423	2.1137	0.7031	2.1785	0.6920
(13, 8)	2.0691	0.4745	1.7849	0.4093	2.0568	0.6286	2.3528	1.0688	2.0946	0.7604	2.1656	0.7238
(13, 7)	2.0066	0.4854	1.6783	0.4060	2.0253	0.6593	2.4343	1.2616	2.0873	0.8407	2.1482	0.7636
(13, 6)	1.9644	0.5070	1.5756	0.4066	1.9994	0.7038	2.6064	1.5999	2.1029	0.9618	2.1213	0.8144
(13, 5)	1.9183	0.5368	1.4765	0.4131	1.9170	0.7456	2.9172	2.2357	2.1591	1.1607	2.0943	0.8889
(13, 4)	1.8444	0.5731	1.3478	0.4188	1.8726	0.8326	3.5988	3.6871	2.3009	1.5336	2.0799	1.0128
(13, 3)	1.8303	0.6573	1.2126	0.4354	1.8225	0.9826	5.3348	7.9824	2.6443	2.3704	2.0837	1.2422
(13, 2)	1.9236	0.8650	1.0610	0.4771	1.8310	1.3410	10.2355	25.6154	3.5913	4.9004	2.2122	1.8218
(10, 10)	2.2760	0.5041	2.1175	0.4690	2.2098	0.6365	2.2769	0.7995	2.2666	0.6768	2.1959	0.6833
(10, 9)	2.2395	0.5120	2.0526	0.4692	2.2026	0.6591	2.2803	0.8646	2.2217	0.7072	2.1954	0.7153
(10, 8)	2.2415	0.5329	1.9579	0.4654	2.1440	0.6707	2.3159	0.9623	2.1916	0.7542	2.1907	0.7495
(10, 7)	2.1343	0.5323	1.8743	0.4675	2.0878	0.6898	2.3645	1.0985	2.1680	0.8197	2.1777	0.7877
(10, 6)	2.0524	0.5432	1.7583	0.4654	2.0607	0.7294	2.4632	1.3176	2.1550	0.9143	2.1550	0.8343
(10, 5)	1.9980	0.5700	1.6460	0.4696	1.9943	0.7718	2.6252	1.6912	2.1665	1.0632	2.1345	0.9021
(10, 4)	1.9024	0.5987	1.4955	0.4706	1.9433	0.8482	3.0057	2.4928	2.2383	1.3319	2.0875	0.9951
(10, 3)	1.8812	0.6786	1.3386	0.4828	1.8753	0.9741	3.9024	4.5663	2.4501	1.9084	2.0637	1.1757
(10, 2)	1.9310	0.8615	1.1825	0.5276	1.8605	1.2789	6.4772	12.2147	3.0365	3.5056	2.1346	1.6284
(8, 8)	2.2995	0.5697	2.1533	0.5335	2.1746	0.7013	2.2975	0.9053	2.2588	0.7561	2.2209	0.7738
(8, 7)	2.2262	0.5747	2.0221	0.5220	2.1427	0.7258	2.3171	1.0080	2.2034	0.8013	2.2183	0.8190
(8, 6)	2.1257	0.5789	1.9308	0.5258	2.1141	0.7609	2.3651	1.1648	2.1629	0.8739	2.2131	0.8716
(8, 5)	1.9916	0.5812	1.7636	0.5147	2.0687	0.8063	2.4651	1.4274	2.1473	0.9920	2.1933	0.9356
(8, 4)	1.9757	0.6321	1.6473	0.5270	2.0084	0.8730	2.7000	1.9523	2.1614	1.1922	2.1756	1.0336
(8, 3)	1.8919	0.6887	1.4724	0.5360	1.9254	0.9813	3.2974	3.2627	2.2803	1.6130	2.1362	1.1907
(8, 2)	1.8515	0.8250	1.2734	0.5674	1.8478	1.2202	4.8599	7.5339	2.6871	2.7558	2.1282	1.5477
(5, 5)	2.2399	0.7026	2.1225	0.6658	2.1963	0.8985	2.3333	1.1728	2.2938	0.9776	2.1842	0.9653
(5, 4)	2.2163	0.7487	1.9756	0.6674	2.1275	0.9491	2.4133	1.4414	2.2179	1.0938	2.1827	1.0632
(5, 3)	2.1206	0.8006	1.7645	0.6662	2.0920	1.0612	2.6252	2.0175	2.2229	1.3548	2.1575	1.1976
(5, 2)	1.9983	0.9030	1.5282	0.6906	1.9827	1.2486	3.2751	3.7155	2.3529	1.9900	2.1385	1.4735

Table 10. BLUE and standard errors for six different coherent systems ($\lambda = 8$).

	System 1		System 2		System 3		System 4		System 5		System 6	
(m, r)	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE
(13, 13)	9.2140	1.7891	8.5376	1.6578	8.7341	2.2032	8.9459	2.7438	8.9174	2.3286	8.7669	2.3886
(13, 12)	9.0731	1.8032	8.2857	1.6467	8.6826	2.2538	8.9323	2.9012	8.7736	2.4021	8.7712	2.4756
(13, 11)	8.7747	1.7929	8.0167	1.6380	8.5231	2.2833	8.9528	3.1048	8.6459	2.5027	8.7700	2.5661
(13, 10)	8.5037	1.7949	7.7014	1.6255	8.4661	2.3526	9.0285	3.3765	8.5280	2.6330	8.7310	2.6554
(13, 9)	8.3382	1.8283	7.4101	1.6248	8.3212	2.4146	9.1477	3.7371	8.4543	2.8124	8.6793	2.7569
(13, 8)	8.3854	1.9229	7.0575	1.6184	8.2059	2.5079	9.3849	4.2632	8.4049	3.0512	8.6174	2.8802
(13, 7)	8.0015	1.9357	6.7369	1.6298	8.0876	2.6326	9.7425	5.0489	8.3940	3.3806	8.5441	3.0371
(13, 6)	7.8121	2.0162	6.3254	1.6325	7.8473	2.7621	10.3483	6.3523	8.4611	3.8698	8.4070	3.2278
(13, 5)	7.7507	2.1688	5.9378	1.6615	7.6094	2.9597	11.5874	8.8806	8.6480	4.6489	8.3434	3.5411
(13, 4)	7.3593	2.2865	5.4015	1.6782	7.3678	3.2759	14.5505	14.9078	9.1798	6.1188	8.2614	4.0226
(13, 3)	7.2839	2.6157	4.8020	1.7244	7.2616	3.9151	21.5748	32.2821	10.6421	9.5397	8.2920	4.9433
(13, 2)	7.8391	3.5252	4.2264	1.9006	7.4121	5.4285	41.0620	102.7617	14.4320	19.6930	8.7559	7.2105
(10, 10)	9.1538	2.0275	8.5204	1.8872	8.7970	2.5339	9.1745	3.2217	8.9706	2.6786	8.8612	2.7574
(10, 9)	9.2002	2.1032	8.1463	1.8623	8.7158	2.6082	9.1584	3.4726	8.8108	2.8045	8.8704	2.8902
(10, 8)	8.7575	2.0818	7.8721	1.8713	8.5707	2.6813	9.2639	3.8495	8.6539	2.9780	8.8431	3.0253
(10, 7)	8.4359	2.1041	7.4636	1.8616	8.3691	2.7651	9.4765	4.4025	8.5532	3.2339	8.7715	3.1728
(10, 6)	7.9623	2.1075	7.0337	1.8617	8.1682	2.8911	9.8835	5.2866	8.4954	3.6044	8.6818	3.3611
(10, 5)	8.1164	2.3155	6.5985	1.8825	8.0612	3.1195	10.5491	6.7961	8.5910	4.2160	8.5944	3.6324
(10, 4)	7.7433	2.4368	6.0344	1.8990	7.7928	3.4013	11.9827	9.9380	8.8557	5.2695	8.4995	4.0516
(10, 3)	7.6019	2.7421	5.3691	1.9367	7.4785	3.8845	15.6528	18.3157	9.6851	7.5438	8.4429	4.8100
(10, 2)	7.5979	3.3896	4.7391	2.1142	7.4193	5.0999	25.7151	48.4935	12.3506	14.2590	8.6045	6.5640
(8, 8)	9.1775	2.2737	8.5250	2.1121	8.7265	2.8142	9.2553	3.6471	9.0602	3.0330	8.8276	3.0756
(8, 7)	8.9363	2.3071	8.1186	2.0960	8.6939	2.9448	9.3309	4.0594	8.8368	3.2139	8.8201	3.2565
(8, 6)	8.6606	2.3586	7.7002	2.0971	8.5116	3.0635	9.5357	4.6963	8.6937	3.5127	8.7937	3.4634
(8, 5)	8.2076	2.3953	7.1365	2.0827	8.2766	3.2260	9.9303	5.7501	8.6521	3.9973	8.7302	3.7239
(8, 4)	8.0653	2.5804	6.5865	2.1073	7.9971	3.4763	10.8698	7.8594	8.6657	4.7799	8.6133	4.0921
(8, 3)	7.5806	2.7596	5.9174	2.1541	7.7278	3.9388	13.0188	12.8818	8.9944	6.3621	8.5275	4.7532
(8, 2)	7.5419	3.3608	5.1321	2.2869	7.6173	5.0299	19.2779	29.8847	10.5515	10.8212	8.6225	6.2708
(5, 5)	9.4185	2.9545	8.5431	2.6799	8.7206	3.5674	9.4915	4.7706	9.0517	3.8580	8.7714	3.8765
(5, 4)	8.7648	2.9609	7.8803	2.6621	8.4787	3.7823	9.7665	5.8334	8.7690	4.3246	8.7334	4.2543
(5, 3)	8.4281	3.1820	7.0079	2.6458	8.3111	4.2159	10.5782	8.1294	8.7161	5.3123	8.6162	4.7828
(5, 2)	7.7591	3.5063	6.1289	2.7696	7.8148	4.9213	13.1145	14.8780	9.3290	7.8902	8.4820	5.8443

Table 11. BLUE and standard errors for six different coherent systems ($\lambda = 15$).

	System 1		System 2		System 3		System 4		System 5		System 6	
(m, r)	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE	$\hat{\lambda}$	SE
(13, 13)	17.4442	3.3872	16.0527	3.1170	16.4641	4.1531	16.8148	5.1574	16.6749	4.3543	16.4707	4.4875
(13, 12)	16.5242	3.2841	15.3804	3.0567	16.1939	4.2035	16.7917	5.4540	16.3784	4.4842	16.4667	4.6476
(13, 11)	16.5698	3.3856	15.0520	3.0755	16.0955	4.3120	16.8618	5.8477	16.1731	4.6816	16.4474	4.8125
(13, 10)	16.0959	3.3973	14.4685	3.0539	16.0842	4.4695	16.9508	6.3393	15.9794	4.9336	16.4175	4.9931
(13, 9)	16.0489	3.5190	13.8390	3.0344	15.6027	4.5275	17.2396	7.0428	15.7707	5.2462	16.3250	5.1856
(13, 8)	15.2346	3.4934	13.2817	3.0456	15.5184	4.7428	17.5811	7.9864	15.6785	5.6918	16.2545	5.4327
(13, 7)	15.1039	3.6539	12.7004	3.0725	15.0609	4.9025	18.3161	9.4921	15.7152	6.3291	16.1122	5.7272
(13, 6)	14.7339	3.8027	11.9447	3.0829	14.7503	5.1919	19.3774	11.8947	15.8272	7.2388	15.8849	6.0988
(13, 5)	14.6318	4.0943	11.0575	3.0941	14.5113	5.6443	21.7467	16.6667	16.2605	8.7411	15.7175	6.6709
(13, 4)	13.7372	4.2680	10.1731	3.1607	14.0158	6.2318	26.7467	27.4035	17.1998	11.4646	15.5714	7.5821
(13, 3)	14.1574	5.0841	9.2264	3.3133	13.6019	7.3334	39.1813	58.6266	19.7780	17.7291	15.6378	9.3225
(13, 2)	14.3741	6.4640	7.9160	3.5598	13.8861	10.1701	76.2287	190.7696	26.7346	36.4805	16.4599	13.5548
(10, 10)	17.3521	3.8434	16.0237	3.5492	16.3785	4.7177	17.1407	6.0190	16.8435	5.0294	16.5292	5.1435
(10, 9)	16.4846	3.7684	15.5015	3.5437	16.1695	4.8387	17.2581	6.5437	16.5215	5.2589	16.5269	5.3848
(10, 8)	16.4691	3.9150	14.7510	3.5066	16.0518	5.0217	17.3941	7.2279	16.2548	5.5936	16.5339	5.6564
(10, 7)	15.7752	3.9346	13.9881	3.4889	15.9560	5.2717	17.7199	8.2322	16.0792	6.0794	16.4418	5.9473
(10, 6)	15.5558	4.1174	13.2482	3.5066	15.4743	5.4771	18.3417	9.8108	16.0040	6.7901	16.3145	6.3160
(10, 5)	15.2284	4.3445	12.3873	3.5340	14.8262	5.7374	19.6518	12.6604	16.1144	7.9081	16.1290	6.8169
(10, 4)	14.2446	4.4826	11.2238	3.5320	14.5792	6.3634	22.4882	18.6508	16.6272	9.8938	15.9881	7.6213
(10, 3)	14.0931	5.0835	10.0982	3.6425	14.0787	7.3128	29.3232	34.3119	18.0935	14.0931	15.9079	9.0629
(10, 2)	14.5898	6.5089	8.6816	3.8731	13.5920	9.3429	48.8040	92.0348	22.3169	25.7651	16.1871	12.3485
(8, 8)	17.0415	4.2220	15.9502	3.9516	16.4881	5.3172	17.3136	6.8225	16.9076	5.6599	16.4164	5.7196
(8, 7)	16.7687	4.3291	15.3147	3.9537	16.1765	5.4793	17.4104	7.5744	16.5036	6.0022	16.4039	6.0566
(8, 6)	16.0655	4.3753	14.4142	3.9256	16.0583	5.7796	17.7471	8.7405	16.1641	6.5312	16.3677	6.4464
(8, 5)	15.8651	4.6300	13.3684	3.9014	15.6777	6.1107	18.4842	10.7031	16.1044	7.4403	16.2166	6.9173
(8, 4)	14.9828	4.7935	12.2562	3.9212	14.8285	6.4459	20.3452	14.7106	16.3369	9.0112	16.0318	7.6165
(8, 3)	14.8166	5.3937	11.1474	4.0580	14.5120	7.3966	24.5948	24.3361	17.2992	12.2365	15.7871	8.7996
(8, 2)	14.3434	6.3916	9.5696	4.2643	13.9960	9.2420	36.4096	56.4424	19.9833	20.4940	15.8119	11.4994
(5, 5)	17.5404	5.5022	15.9798	5.0127	16.3820	6.7016	17.5799	8.8361	16.9298	7.2158	16.5034	7.2937
(5, 4)	16.7258	5.6503	14.8591	5.0196	15.9796	7.1284	18.0264	10.7669	16.4483	8.1119	16.5032	8.0391
(5, 3)	15.8518	5.9848	13.4068	5.0617	15.5838	7.9050	19.5587	15.0309	16.5053	10.0596	16.4313	9.1209
(5, 2)	14.5472	6.5738	11.3400	5.1245	14.8484	9.3507	24.2085	27.4639	17.6925	14.9639	16.2433	11.1920

Table 12. BLUP and variances for six different coherent systems ($\lambda = 0.5, m = 13, r = 7$).

Samples & Predictors & V	$T_{1:13}$	$T_{2:13}$	$T_{3:13}$	$T_{4:13}$	$T_{5:13}$	$T_{6:13}$	$T_{7:13}$	$T_{8:13}$	$T_{9:13}$	$T_{10:13}$	$T_{11:13}$	$T_{12:13}$	$T_{13:13}$
System 1 Samples	0.3407	0.7129	1.1126	1.5459	2.0371	2.5915	3.2190	3.9647	4.8380	5.9396	7.4051	9.6377	14.0324
System 1 Predictors								3.8202	4.5110	5.3418	6.4084	7.9503	10.9016
V								0.0151	0.0210	0.0283	0.0374	0.0498	0.0717
System 2 Samples	0.7367	1.5174	2.3609	3.3044	4.3535	5.5278	6.8753	8.4759	10.3708	12.7609	15.8790	20.6649	30.0363
System 2 Predictors								7.7956	8.8478	10.1063	11.7347	14.0834	18.6752
V								0.0334	0.0432	0.0546	0.0697	0.0891	0.1276
System 3 Samples	0.4625	0.9637	1.5182	2.1232	2.8066	3.5769	4.4341	5.4497	6.6892	8.2383	10.2141	13.2320	19.2287
System 3 Predictors								5.2804	6.2761	7.5084	9.1455	11.6252	16.6908
V								0.0294	0.0415	0.0570	0.0779	0.1096	0.1743
System 4 Samples	0.1786	0.3732	0.5874	0.8214	1.0807	1.3719	1.7109	2.0983	2.5718	3.1596	3.9513	5.1172	7.4762
System 4 Predictors								2.1761	2.7258	3.4013	4.2855	5.5839	8.1129
V								0.0107	0.0174	0.0264	0.0385	0.0553	0.0841
System 5 Samples	0.3080	0.6401	1.0094	1.4059	1.8547	2.3578	2.9237	3.5860	4.3801	5.3853	6.7070	8.7060	12.6795
System 5 Predictors								3.5871	4.3524	5.2690	6.4451	8.1303	11.3218
V								0.0198	0.0294	0.0415	0.0566	0.0772	0.1106
System 6 Samples	0.4301	0.8960	1.4140	1.9945	2.6211	3.3267	4.1466	5.0992	6.2348	7.6710	9.5387	12.3637	17.9859
System 6 Predictors								5.0204	6.0709	7.3910	9.1760	11.8929	17.4543
V								0.0320	0.0466	0.0661	0.0936	0.1365	0.2264

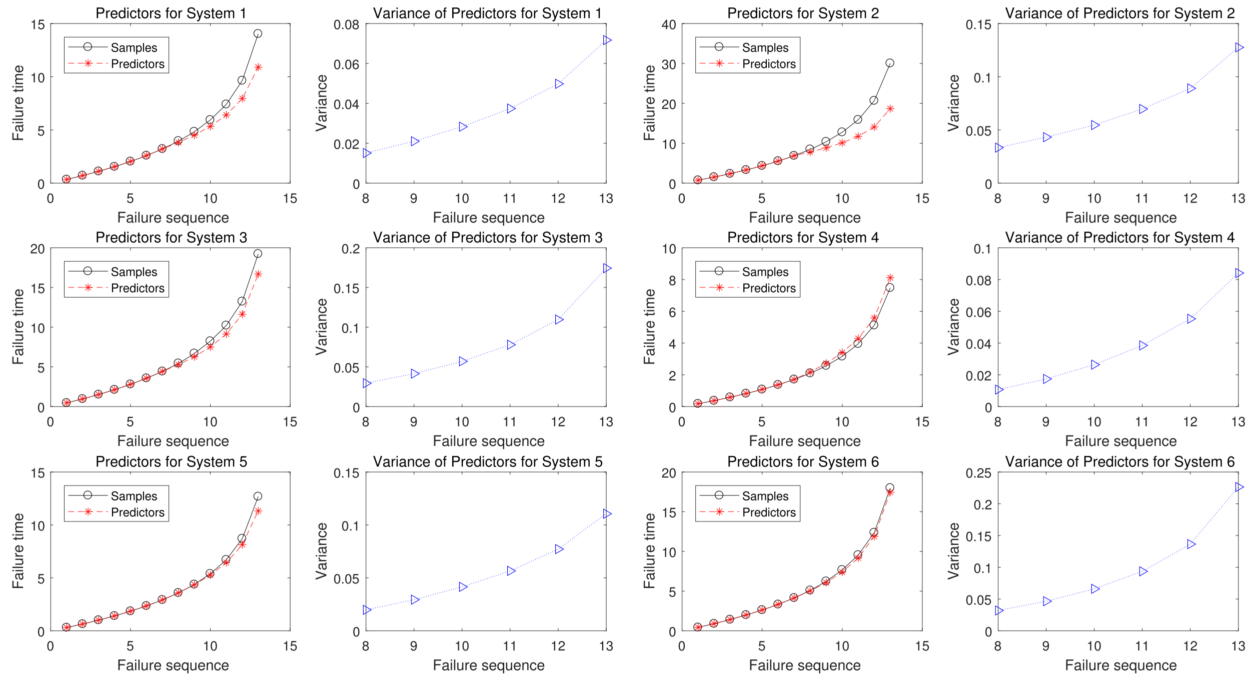

Figure 1. Comparison of the BLUP and samples and variation of variances.

Table 13. BLUE, K–S distance, and p-values of $F_{T^{(1)}}$ and $F_{T^{(2)}}$ based on data set.

Data set	$\hat{\lambda}_1$	$\hat{\lambda}_2$	K–S	p-value
$T^{(1)}$	8.3336		0.1504	0.9120
$T^{(2)}$		7.3822	0.1632	0.8566

Table 14. BLUP and variances for real data analysis($(m, r) = (12, 8)$).

Predictors & V	$\hat{T}^{(1)}_{9:12}$	$\hat{T}^{(1)}_{10:12}$	$\hat{T}^{(1)}_{11:12}$	$\hat{T}^{(1)}_{12:12}$
System 1 Predictors	90.0	108.9	136.1	188.2
V	7.2266	10.5089	15.3577	24.7440
System 3 Predictors	103.8	127.5	163.3	236.3
V	10.4172	15.5148	23.7706	42.1318

represents failure times, in minutes, for two types of electrical insulation in an experiment in which the insulation is subjected to a continuously increasing voltage stress. 12 electrical insulations of each type are tested and recorded. The failure times of the first type $T^{(1)}$ are 12.3, 21.8, 24.4, 28.6, 43.2, 46.9, 70.7, 75.3, 95.5, 98.1, 138.6 and 151.9, while the failure times of the second type $T^{(2)}$ are 18.5, 21.7, 35.1, 40.5, 42.3, 48.7, 79.4, 86.0, 121.9, 147.1, 150.2 and 219.3.

Let $T^{(1)}$ and $T^{(2)}$ denote the failure times of two different coherent systems. It is assumed that the failure data $T^{(1)}$ and $T^{(2)}$ are from System 1 and System 3, respectively. Fix $\alpha = 1$, $\beta = 9$ and $c = 0.1$. Based on Equation (12), the CDFs of $T^{(1)}$ and $T^{(2)}$ can be expressed as$F_{T^{(1)}} = 1 - R_{T^{(1)}} = 1 - \sum_{j=1}^{3} a_j^{(1)} b_j e^{-((\sigma_j t)/\lambda)}$, $t \geq 0$ and $F_{T^{(2)}} = 1 - R_{T^{(2)}} = 1 - \sum_{j=1}^{3} a_j^{(2)} b_j e^{-(\sigma_j t/\lambda)}$, $t \geq 0$, where $\mathbf{a}^{(1)} = (a_1^{(1)}, a_2^{(1)}, a_3^{(1)}) = (0, 3, -2)$ and $\mathbf{a}^{(2)} = (a_1^{(2)}, a_2^{(2)}, a_3^{(2)}) = (1, 1, -1)$. Using the proposed method in Section 3 and fixing $m_1 = r_1 = m_2 = r_2 = 12$, the BLUE of λ_1 for $T^{(1)}$ and λ_2 for $T^{(2)}$ can be computed, respectively. The

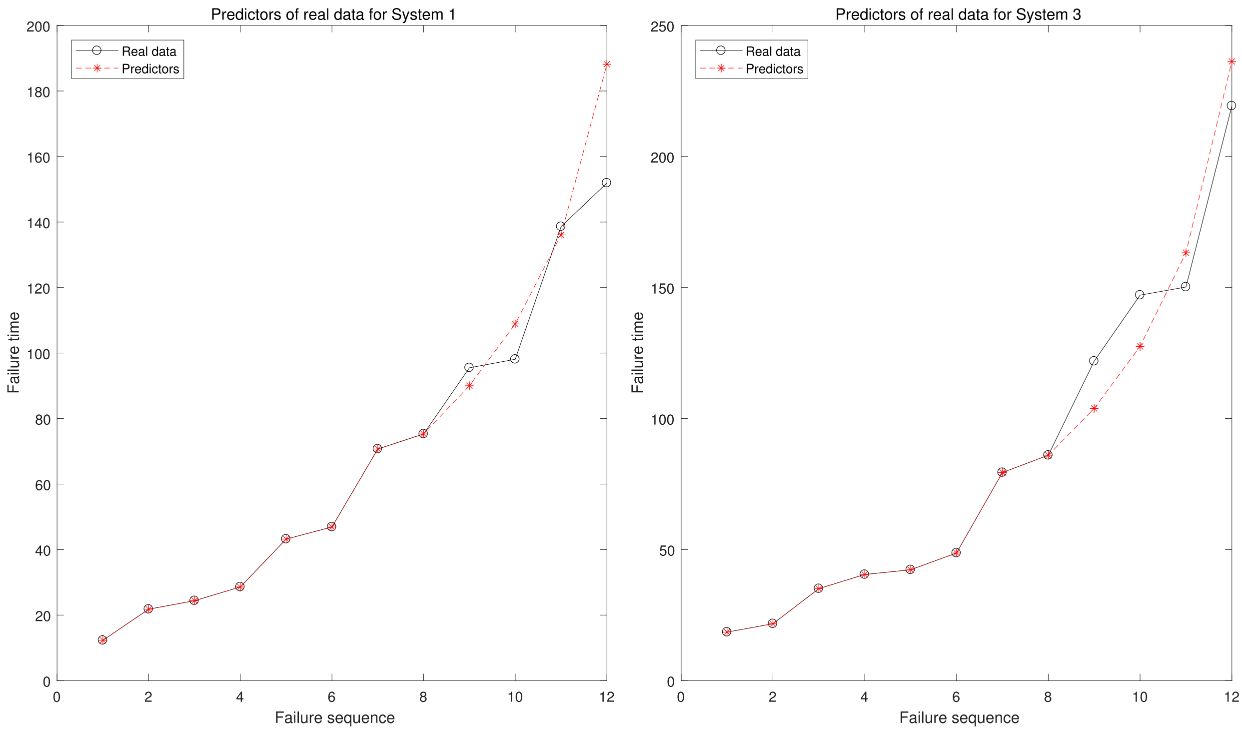

Figure 2. Comparison of the BLUP and real data samples.

Kolmogorov–Smirnov (K–S) test statistic and corresponding p-values are computed by using $\hat{\lambda}_1$ and $\hat{\lambda}_2$. The BLUEs of λ_1 and λ_2, K–S distances between the empirical distribution functions and the fitted distribution functions and corresponding p-values are presented in Table 13. Then, based on the K–S distances and p-values, one cannot reject the hypothesis that the data are coming from the above distributions.

For illustrative purpose, let $m_1 = m_2 = m = 12$ and $r_1 = r_2 = r = 8$. It is supposed that $T^{(1)}_{1:m}, T^{(1)}_{2:m}, \ldots, T^{(1)}_{r:m}$ are the observed failure times for System 1 and $T^{(2)}_{1:m}, T^{(2)}_{2:m}, \ldots, T^{(2)}_{r:m}$ are the observed failure times for System 3. According to the proposed method in Section 4, the predictors $\hat{T}^{(1)}_{r+1:m}, \hat{T}^{(1)}_{r+2:m}, \ldots, \hat{T}^{(1)}_{m:m}$ and $\hat{T}^{(2)}_{r+1:m}, \hat{T}^{(2)}_{r+2:m}, \ldots, \hat{T}^{(2)}_{m:m}$ of the Type-II censored (unobserved) system failure times for System 1 and System 3 can be computed, respectively. For each coherent system, the predictors and corresponding variance V are presented in Table 14, as well as the comparisons of the predictors and the last $m-r$ samples are shown in Figure 2.

7. Conclusion

In this paper, we study the BLUE for dynamic stress–strength reliability of coherent systems, which consist of multiple identical components with random strength and are subjected to repeated random stresses at random cycle times, with survival signature. In addition, the BLUP for the Type-II censored coherent (unobserved) system failure times based on the observed failure times are discussed. According to the results of numerical simulation, it is readily seen that the larger the sample size, the more accurate the estimation. The failure number $r = [m/2]$ may minimise the deviation between the BLUE and parameter λ. In addition, the BLUP has a good performance.

Disclosure statement

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