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SHORT COMMUNICATION

Responses to discussions on 'Optimal reinsurance designs based on risk measures: a review'

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We would like to thank four discussants (Professor Chengguo Weng, Professor Tim J. Boonen, Professor Shengchao Zhuang, and Professor Ambrose Lo) for their interesting discussions and comments on our review paper about optimal reinsurance designs based on risk measures. Their discussions and comments provide more other interesting research topics for the study of optimal reinsurance designs. Our specified response to each discussant is given as follows.

1. Response to Chengguo Weng

Thank Chengguo for bringing the marginal indemnification function (MIF) approach and the empirical approach to our attention. It has been illustrated in several research papers that these two approaches are very powerful in solving many optimal reinsurance problems. However, it is worth noting that there are some limitations to their applications.

As Chengguo mentioned in his discussion, the application of the MIF approach relies heavily on the assumption of ceded loss functions. To use this approach, one has often to assume that the marginal ceded loss function is bounded from below and above. This assumption is consistent with insurance practice and can reduce the ex-post moral hazard. In addition, the usefulness of this approach also depends on the optimisation criterion and the reinsurance premium principle. For optimal reinsurance problems with distortion risk measures and distorted reinsurance premium principles, the objective function is linear in the marginal ceded loss function and hence this approach is very powerful in deriving the solutions to such problems. However, this approach may fail if the objective function is nonlinear in the marginal ceded loss function.

Up to the present, an analytical optimal solution is often obtained by imposing very stringent assumptions in an optimal reinsurance model. If more practical constraints are added to an optimal reinsurance model, it will be very challenging to solve such an infinite-dimensional optimal reinsurance problem. Interestingly, it is possible that one first reduces the dimension of the problem by focusing on the empirical loss distribution, then the optimal reinsurance design under a variance or Conditional Value at Risk (CVaR) is reduced to a second-order conic programming problem. For second-order conic programming problems, many existing numerical approaches can be used. Unfortunately, it is worth noting that under Value at Risk (VaR), the problem cannot be transformed into a second-order conic programming problem. Furthermore, it is not an easy task to analyse the consistency and convergence rate of the solutions obtained from an empirical model.

2. Response to Tim J. Boonen

In many optimal reinsurance problems, the objectives of finding optimal reinsurance strategies are to minimise some risk measure of an insurer's risk exposure, and yet the insurer's profit is often neglected. Under such a criterion, the insurer would not like to take any risk, as pointed out by Tim. In reinsurance practice, the insurer would like to retain more risk such that more expected profit can be obtained, but the cost of capital incentives the insurer to cede part of risk to a reinsurer. In other words, reducing the cost of capital is one of the purposes of reinsurance due to the regulation. Thus, the criterion of minimising the risk-adjusted liability of the insurer, which is composed of the best estimate to the retained risk and the capital cost, has been introduced to analyse the optimal reinsurance design. See Section 4 of our review paper for detailed discussions.

Tim further pointed out some differences between the optimal reinsurance design and the risk-sharing problem. More specifically, while the optimal reinsurance problem can be treated as a special case of risk

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sharing, the reinsurer can benefit from diversification by signing many similar contracts. Thus, it is quite necessary to take into consideration other risks the reinsurer is facing. In previous studies, this problem is partly solved by using the premium principle to price the reinsurance contract and the retrocession. More specifically, when the reinsurer has lots of independent and identically distributed (i.i.d.) risks, the expected value premium principle can be used to price the contract based on the Strong Law of Large number. However, when the i.i.d. assumption does not hold or the number of homogeneous contracts the reinsurer signs is finite, other risks have to be incorporated in the reinsurer's risk exposure and the optimal reinsurance design may be quite different. For this case, the optimal solutions are unclear, and it deserves more attention in future research.

In addition, Tim also mentioned the effects of the asymmetric information and the capital constraint on the optimal reinsurance design. These effects are very important and have been briefly discussed in our review paper.

3. Response to Shengchao Zhuang

Shengchao presented an interesting hybrid model of optimal reinsurance in his discussion. Different from other studies, he considered the risk allocation among three agents: an insured, an insurer and a reinsurer. The problem was analysed from the perspective of an insurer, who can decide the reinsurance strategy, the amount of insurance premium and the risk covered for an insured and would like to minimise the risk exposure under distortion risk measures. When the insurance premium and the reinsurance premium are calculated by distorted premium principles, he used the MIF approach to derive optimal solutions explicitly. The effects of the incentive condition, the insurance premium budget constraint and the reinsurance premium budget constraint were also investigated.

Notably, in the hybrid model of optimal reinsurance Shengchao discussed, only the single risk of the insured is considered. The diversification benefit of the insurer is completely neglected, as pointed out by Tim. Therefore, this hybrid model becomes a pure risk-sharing problem and may not be consistent with insurance practice. The results may be more interesting if the diversification effect can be taken into consideration.

4. Response to Ambrose Lo

Thank Ambrose for highlighting the importance of practical constraints in the design of an optimal reinsurance contract. In his discussion, Ambrose also pointed out the technical challenges in solving optimal reinsurance problems with these practical constraints. More specifically, under distortion risk measures and distorted premium principles, optimal reinsurance problems with a single external constraint can often be solved by a Neyman–Pearson approach. However, for multiple external constraints, this approach may not work except for very special cases. Thus, more sophisticated techniques are needed to solve such problems.

In practice, when signing a reinsurance contract, the insurer and the reinsurer need to consider the insurance regulation, the insurer's expected profit, and the reinsurer's risk tolerance. Naturally, some practical constraints often appear in the negotiation of a reinsurance contract. Thus, it is of practical interest to investigate optimal reinsurance problems with these constraints. Obviously, different constraints may lead to different optimal reinsurance models. Interestingly, some reinsurance models have a very similar mathematical structure, and it motivates one to use a common approach to solve these problems. As discussed by Ambrose, this line of research is far from complete and many technical challenges are unresolved. We expect some breakthroughs can be made in near future.

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