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Application of autoregressive tail-index model to China’s stock market

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Motivated by Section 5.1 in Zhang (2020), we first fit a GARCH(1,1) model with normal distributed innovations to each individual negative log-retuens series. Using the negative log-retuens series divided by the fitted volatilities, we obtain standardised negative log-retuens series for each stock. Taking the maximum value of the 87 standardised negative log-retuens each day, we obtain a time series \( \{Q_t: t = 1, 2, \ldots, 3836\} \), see Figure 3. It is seen that there exist four possible peaks around June 2006, November 2008, January 2016 and September 2018. In fact, China’s stock market experienced substantial boom and burst during these five periods. On June 7, 2006, SSE Index plummeted 88.45 points. In 2008, the US subprime mortgage crisis spread to the world and triggered a financial tsunami. SSE Index dropped from the highest of 6124 in October 2007 to the lowest of 1664 in October 2008. By the end of 2015, SSE Index was up 12.6% rebounding to 3600. In January 2016, China’s stock market experienced a steep sell-off and trading was halted on January 7, 2016 after the market fell 7%. On January 26, 2016, SSE Index fell below the lowest point in August 2015, and on January 27, it fell down to 2638. After SSE Index fell below the 2016 lowest point 2638 on October 11, 2018, it slid below 2500 on October 18, and fell to 2449 on October 19, which plunged more than half from the 2015 highest point. The performance of \( \{Q_t\} \) series is consistent with the empirical observations related to China’s stock market. Figure 4 presents the histogram of \( \{Q_t\} \) series, which indicates that the standardised negative log-retuens possibly follow Fréchet distribution.

We fit \( \{Q_t\} \) by the autoregressive tail-index model, i.e. model (5.1)-(5.3) in Zhang (2020). The fitted parameter values and standard deviations are presented in Table 1. It is shown that all parameters are significant, which indicates model (5.1)-(5.3) is suitable for the cross-sectional maxima of 87 stocks in SSE Index.

The recovered tail indices \( \{\hat{\alpha}_t\} \) are presented in Figure 5. Obviously, when the extreme events appear, the tail index tends to decrease, reflecting an increase
Figure 1. Daily closing prices of SSE Index from 01/05/2005 to 10/20/2020.

Figure 2. Histogram of daily maxima of negative log-returns of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

Figure 3. Daily maxima of standardised negative log-returns of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.
in risk. Figure 6 shows the recovered scale parameters \( \{\hat{\sigma}_t\} \).

Based on the recovered \( \{\hat{\alpha}_t\} \) and \( \{\hat{\sigma}_t\} \), we estimate the 90% confidence interval of \( \{Q_t\} \) by (5.1), which is presented in Figure 7. We find that 90.6% of \( \{Q_t\} \) lie in the estimated 90% confidence interval.

We further test the out-of-sample performance of model (5.1)–(5.3) for predicting 1-day conditional Value at Risk (conditional VaR) for daily maxima negative log-returns of SSE Index. Conditional VaR is the most commonly used measure for tail risk in financial applications. For \( 0 \leq q \leq 1 \), conditional VaR(q) is defined as the \( 1-q \) extreme quantile of \( \{Q_t\} \) given all past information \( F_{t-1} \). First, we fit model (5.1)–(5.3) using the 2500 observations where \( 1 \leq t \leq 2500 \) (roughly 10 years). For the rest 1336 observations where \( 2501 \leq t \leq 3836 \), based on the fitted model (5.1)–(5.3) and past information \( F_{t-1} \), we calculate their 1-day conditional VaR(q) at \( q^0 = 0.1, 0.05, 0.01, 0.005, 0.001 \). The true daily maxima are then compared with the estimated conditional VaR and the number of violations is recorded. Table 2 presents the expected violations (1336\( q^0 \)), the number of actual violations, the violation rates and relative error rate. It is clear that the 1-day conditional VaR based on model (5.1)–(5.3) performs well, with small relative error rate and violation rate close to \( q^0 \).

We also use the autoregressive conditional Weibull model (i.e. \( Y_t \) in model (5.1) is a unit Weibull random variable), to fit China’s stock market. It does not perform as well as model (5.1)–(5.3) with unit Fréchet random variable, so we do not present the results.

**Table 1.** MLE for cross-sectional daily maxima of negative log-returns of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

<table>
<thead>
<tr>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-7.548</td>
<td>0.794</td>
<td>8.297</td>
<td>0.007</td>
<td>0.579</td>
<td>0.804</td>
<td>-0.021</td>
<td>0.933</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.333</td>
<td>0.006</td>
<td>0.351</td>
<td>0.001</td>
<td>0.080</td>
<td>0.026</td>
<td>0.004</td>
<td>0.167</td>
</tr>
</tbody>
</table>

**Figure 4.** Histogram of daily maxima of standardised negative log-returns of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

**Figure 5.** Recovered tail indexes \( \{\hat{\alpha}_t\} \) (dashed), and daily maxima of standardised negative log-returns (solid) of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

**Figure 6.** Recovered scale parameters \( \{\hat{\sigma}_t\} \).
Figure 6. Recovered scale parameters \( \hat{\sigma}_t \) of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

Figure 7. The 5% quantile (dashed), 95% quantile (dash-dotted) of the estimated \( Q_t \) and daily maxima of standardised negative log-returns (solid) of 87 stocks in SSE Index from 01/05/2005 to 10/20/2020.

Table 2. The performance of model (5.1)–(5.3) on approximation of 1-day conditional VaR for daily maxima negative log-returns of SSE Index.

<table>
<thead>
<tr>
<th>( q )</th>
<th>Expected violation</th>
<th>Actual violation</th>
<th>Violation rate</th>
<th>Relative error rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>133.6</td>
<td>110</td>
<td>0.082</td>
<td>-0.177</td>
</tr>
<tr>
<td>0.05</td>
<td>66.8</td>
<td>57</td>
<td>0.043</td>
<td>-0.147</td>
</tr>
<tr>
<td>0.01</td>
<td>13.4</td>
<td>21</td>
<td>0.016</td>
<td>0.567</td>
</tr>
<tr>
<td>0.005</td>
<td>6.7</td>
<td>15</td>
<td>0.011</td>
<td>1.239</td>
</tr>
<tr>
<td>0.001</td>
<td>1.3</td>
<td>6</td>
<td>0.004</td>
<td>3.015</td>
</tr>
</tbody>
</table>

We believe that the autoregressive tail-index model (5.1)–(5.3) also captures the systematic risk in China’s stock market.

In a summary, although the autoregressive tail-index model presented in Zhang (2020) is novel and advanced, it cannot be directly applied to the stock data from China’s market. However, it can be applied to pseudo-returns which we studied and discussed.

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