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Rejoinder of "On studying extreme values and systematic risks with nonlinear time series models and tail dependence measures"

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I am pleased that my review article has stimulated such broader and thoughtful discussions in probability theory, theoretical statistics, estimation methods, and applications. The discussants have made many excellent points. I appreciate the discussants' interest in the reviewed contents and much broader theoretical and methodological topics related to extreme value study. In particular, Ji and Li (2021), find a way that one of the reviewed models can be extended to study the systematic risks in the Chinese stock market. Qi (2021) points out that the estimation of the static tail index parameter in the generalised extreme value distribution is still far from perfect, and then discusses three maximum likelihood estimations from Hall (1982), Peng and Qi (2009), and F. Wang et al. (2019) to handle the tail index that falls in different ranges. Smith (2021) offers a much more general view of the development of extreme value theory over the last thirty years. Readers can benefit from reading the discussions and the references discussed therein. T. Wang and Yan (2021) not only extend discussions to two extreme dependence measures introduced by Resnick (2004) and Davis and Mikosch (2009) but also point out some practical issues existed in many extreme value applications. Xu and Wang (2021) show some interesting ideas of extending the tail quotient correlation coefficient to the conditional tail quotient correlation coefficient for conditional tail independence. They also outline some ideas of applying the new extreme value theory for maxima of maxima for high-dimensional inference, e.g., multiple testing problems. T. Zhang (2021a) focuses on time series extremes and advocates measuring the cumulative tail adversarial effect, i.e., the degree of serial tail dependence and the desired limit theorem in T. Zhang (2021b).

My review is focussing on studying extreme values and systematic risks with nonlinear time series models and tail dependence measures, and of course, it is not the final word on the reviewed topics and the topics discussed by the discussants, and many other broad topics researched by the extreme value literature. I look forward to future developments in all of these areas. This rejoinder will further clarify some basic ideas behind each reviewed measures, models, their applications, and their further developments.

Interpretability, computability, and testability. Some basic properties, such as interpretability, computability, predictability, stability, and testability, are often desired in statistical applications. In general, parametric models can satisfy these properties and are widely adopted. For example, linear regressions are the most popular models used daily, and Pearson's linear correlation coefficient is the most commonly used dependence measure between two random variables. On the other hand, parametric models may not be general enough, and their models' assumptions may not be satisfied. As a result, nonparametric (semi-parametric) models, random forest, deep learning models, and neural network models are preferred. However, these general and advanced models bring some difficulties in achieving some or all of the aforementioned desired properties. As to how to choose a model in practice, it depends on many factors. George Box stated that all models are wrong, but some are useful. There is a tradeoff between parametric models and nonparametric models. We may say that all models are useful, but the strengths vary with each individual.

Analogs to linear regression and Pearson's linear correlation coefficient are not yet well defined in the extreme value context. The extreme dependence measures discussed in T. Wang and Yan (2021) and the most popular coefficient of tail dependence measure η by Ledford and Tawn (1996, 1997) often involve nonparametric estimations. The quotient correlation coefficient (QCC) and the tail quotient correlation coefficient (TQCC) were introduced in Z. Zhang (2008) as alternative correlation measures to the linear correlation coefficient (LCC). It can be seen from Examples 3.1 and 3.2 in Z. Zhang (2020) the LCC is an absolute error based measure while QCC/TQCC is a relative error based

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measure. The interpretability, computability, and testability of TQCC can clearly be seen in Equations (18) and (19). TQCC can easily be applied to high-dimensional pair-wise computing as it just needs to first compute marginal transformations to unit Fréchet scales. AS to the marginal transformation, we can also apply the rank-based transformation; see Z. Zhang et al. (2011) for theoretical justification and computational details.

Another popular application of testing and modelling tail independence and tail dependence is through copula construction. The most common copula used in the extreme value literature is the Gumbel copula; see Joe (2014) for dependence modelling and copula and Feng et al. (2018) for a market risk application. In general, a single Gumbel copula is not suitable for high-dimensional modelling. A hierarchical Gumbel copula is ideal for many applications. Joe (2014) reviewed many copula families. There are two flexible copula families, MGB2 (multivariate generalised beta of the second kind, Yang et al., 2011) and Max-Copula (Zhao & Zhang, 2018), that can be used in high-dimensional modelling, especially in dynamic time series models.

Threshold issue. This has been an unavoidable issue in extreme value applications. We hardly find any data directly follow an extreme value type distribution. Under rigorous extreme value theory, both the block maxima and the exceedance values have limit extreme value distributions, i.e., the generalised extreme value (GEV) distribution and the generalised Pareto distribution (GPD). However, in practice, the sizes of blocks in GEV fitting and the threshold levels in GPD fitting are always finite given a finite number of sample data. As discussed in T. Wang and Yan (2021), the threshold levels are often selected as 90%, 95%, 99% percentiles (McNeil & Frey, 2000). In TQCC computations, using Theorem 3.8 in Z. Zhang (2020), a threshold needs to be selected. However, Theorem 3.7 does not involve a concrete threshold as the theory is built on random thresholds. We can simulate a sequence of random thresholds and compute a set of TQCCs. For example, we can choose the mean or the median of these TQCCs as the estimate of the tail index. These TQCCs can be used in testing $\lambda = 0$ defined in (16) in the review paper and predicting the probability of observing a rare event in the future in a time series model. More theoretical developments are needed in this direction.

Estimations in static models and dynamic models. Qi (2021) points out some problems in extreme value distribution inferences and some remedies in estimating the tail index and the endpoint. In the financial risk management and extreme events modelling, the location parameter is also important and has its financial meaning; see Zhao et al. (2018) for discussions. We found that the parameters (location, scale, and shape (tail index)) specified in dynamic time series models may be estimated satisfactorily, e.g., Zhao et al. (2018) and Deng et al. (2020). In Smith (2021), two questions have been raised. The first one is about the exact maximum likelihood (or Bayesian) estimation of the processes reviewed in Z. Zhang (2020). Definitely, many efforts need to put in this direction. Smith (2021) points out the maximum likelihood estimation and Bayesian estimation of Brown-Resnick process (Brown & Resnick, 1977; Kabluchko et al., 2009; Oesting et al., 2017; Schlather, 2002) have achieved significant progress, while their applications in M4 processes and related processes are not well studied. As discussed in T. Wang and Yan (2021), alternative methods for intractable likelihood such as composite likelihood and approximate Bayesian computation in extreme value time series, e.g., M4 process, can be efficient. Other alternative methods, such as particle filtering methods and a new Bayesian framework – Auxiliary Least Squares Optimization, can be efficient as well. There are some considerable progress in this direction, e.g., Kunihama et al. (2012), Idowu and Zhang (2017), and Mao and Zhang (2018). The second question in Smith (2021) is how to model high-dimensional time series using the reviewed time series models and other models, e.g., MARMA, Brown-Resnick type models, given that much work in the literature was still in low-dimensions. One idea is to use copula to link univariate extreme value time series, e.g., copula structured M4 process models in Z. Zhang and Zhu (2016). The max-copula (Zhao & Zhang, 2018) and MGB2 copula (Yang et al., 2011) can be flexible and efficient in linking high-dimension time series. Recently, T. Zhang (2021b) proposed an alternative framework based on a new concept of tail adversarial stability for asymptotic theory of tail dependent time series. Unlike the strong mixing condition, the framework of T. Zhang (2021b) quantifies the degree of serial tail dependence by measuring the cumulative tail adversarial effect of coupled innovations and was shown to coordinate well with a lag-l tail dependent martingale approximation scheme that can help lead to the desired limit theorem. The tail adversarial stability and the lagged tail dependence index may be used to guide the high-dimension extreme value time series building.

Conditional tail independence. The conditional inference is a contemporary research topic in social science, biological science, and statistical science. However, it hasn't drawn much attention in the extreme value studies. Xu and Wang (2021)'s extension from TQCC to a conditional TQCC can be a promising research direction.

Software developments. As discussed in T. Wang and Yan (2021), the availability of software packages for ARMA models and GARCH models makes their applications fruitful. Although there are considerable progresses in extreme value theory and time series building of MARMA (Davis & Resnick, 1989, 1993), M4 (Smith & Weissman, 1996; Tang et al., 2013; Z. Zhang

& Smith, 2004, 2010), $MAP_{\alpha}(1)$ (Naveau et al., 2011), AcF (autoregressive conditional Fréchet, Ji & Li, 2021; Zhao et al., 2018), STI (stochastic tail index, Mao & Zhang, 2018), DCW (dynamic conditional Weibull, Deng et al., 2020), the software packages for these models are not well developed. There is an urgent need in this direction.

Conclusions. Studies on systematic risks have drawn attention in recent years. Extreme value analysis can play a central role in this area. There are many interesting and important research problems awaiting researchers to put effort into. The discussants have identified a number of interesting research topics. The most recent paper (Cao & Zhang, 2020) in new extreme value theory of maxima of maxima for heterogeneous populations can be a starting point for many new research topics in high-dimension extreme value studies.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributor

Zhengjun Zhang is Professor of Statistics at the University of Wisconsin. His main research areas of expertise are in financial time series and rare event modelling, virtual standard cryptocurrency, risk management, nonlinear dependence, asymmetric dependence, asymmetric and directed causal inference, gene-gene relationship in rare diseases.

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