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A discussion of 'optimal reinsurance designs based on risk measures: a review'

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I congratulate the authors on their review article (Cai and Chi, 2020). Their paper reviews the academic literature on optimal reinsurance problems with risk measures.

In optimal reinsurance, an insurer seeks to share the exogenously given risk X with a reinsurer. The insurer cedes the risk $f(X)$, where $0 \leq f(X) \leq X$. Optimal reinsurance problems are formulated as

$$\min_{f \in \mathcal{C}} \rho(X - f(X) + \pi(f(X))), \qquad (1)$$

where ρ is a risk measure such as the Value-at-Risk (VaR) or the conditional Value-at-Risk (CVaR), and π is exogenously given reinsurance premium principle. Moreover, $\mathcal{C}$ is a set of admissible ceded loss functions.

Some problems of this type have been solved by Cai and Tan (2007), and later papers generalise this to include

- several different choices of the set $\mathcal{C}$;
- a risk-sharing reformulation for the insurer and reinsurer by assuming Pareto-optimality;
- multivariate risk, so that the reinsurer can share risk with multiple insurers.

The review article of Cai and Chi (2020) is complete. However, another important type of generalisations has been to impose constraints on the premium (also called a premium-budget). As a discussion, I encourage the authors and interested readers to think about bringing the optimal reinsurance literature closer to the reinsurance industry. In particular, I believe it would be helpful to include (even more) realistic assumptions, which will then help us to understand the reinsurance market even better.

1. Comment 1: cost of capital

The risk measures VaR and CVaR originate from regulation, and are used to determine capital requirements (see, e.g., Solvency II in the European Union or the Swiss Solvency Test regulations). Minimising (1) with ρ being a VaR or CVaR risk measure is generally not a driving criterion in reinsurance practice, as would it be optimal for the insurer to stay out of insurance business altogether resulting in a zero value of this risk measure.[1] In general, it seems implausible that the *only* criterion for the insurer is to minimise a VaR or CVaR risk measure, as such risk measures do not attach any value to 'small' deviations in the risk exposure below a given quantile. However, holding a buffer as capital is costly for the insurer, and this induces a cost-of-capital. A purpose of the reinsurance is to reduce this cost of capital, and thus reinsurance is attractive if the premium is smaller than the cost of capital reduction in reinsurance (see, e.g., Albrecher et al., 2017).

For instance, instead of minimising a risk measure in (1), the insurer may minimise the following objective:

$$\min_{f \in \mathcal{C}} E[X - f(X)] + \pi(f(X))$$
$$+ \delta_{CoC} \cdot (\rho(X - f(X) - E[X - f(X)])), \quad (2)$$

where $\delta_{CoC} \in [0, 1]$ is the cost of holding capital. The insurer is then risk-neutral, but incurs costs of holding a capital buffer. This buffer is determined by a risk measure, such as the VaR or the CVaR. The costs of holding capital make the agent adverse to holding risk. See, for instance, Chi (2012), Cheung and Lo (2017) and also Cai and Chi (2020) for a thorough motivation of this objective.

Then, since the VaR and the CVaR are distortion risk measures (Wang et al., 1997), the objective in (2) is mathematically equivalent to a distortion risk measure (see Boonen et al., 2016a, 2016b). In particular, the objective in (2) can be written as

$$\pi(f(X)) + \int_0^\infty g_\rho(\mathbb{P}(X - f(X) > z)) \, dz, \qquad (3)$$

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[1] This claim is originally posed by Albrecher et al. (2017, p. 262 therein).

where if ρ is a VaR risk measure then $g_{\text{VaR}}(s) = (1 - \delta_{CoC})s + \delta_{CoC}\mathbf{1}_{s>\alpha}$, and if ρ is a CVaR risk measure then $g_{\text{CVaR}}(s) = (1 - \delta_{CoC})s + \delta_{CoC}\min\{s/\alpha, 1\}$, $s \in [0, 1]$. Here, $\mathbf{1}_{s>\alpha} = 1$ if $s > \alpha$, and it is zero otherwise. If we minimise (3) over reinsurance contracts $f \in \mathcal{C}$, with

$$\mathcal{C} = \mathcal{C}_2 := \{f : 0 \le f(x) - f(y) \le x - y, \forall\, 0 \le y \le x\},$$

and if π is a Wang's premium principle, then one can derive the optimal reinsurance indemnity contracts using Theorem 2.1 in Cui et al. (2013). Such an optimal reinsurance indemnity is of a layer-reinsurance type, where there may be many layers.

2. Comment 2: other reasons for reinsurance

Optimal reinsurance contract theory is a classical example of a risk-sharing problem. The insurer shares risk with a reinsurer, and both agents are expected to benefit from such a transaction. A reinsurer should not be treated as another insurer, as many reinsurers do not offer any primary insurance and vice versa.

One wonders how reinsurance companies differ from the insurers in the framework posed by many papers on optimal reinsurance, including the review article of Cai and Chi (2020). Consider the setting where the reinsurance company is modelled as an economic agent in a risk-sharing model. Then, a reason for risk-sharing would be differences in risk-attitudes, such as the results derived in Asimit and Boonen (2018) or Section 6 of Cai and Chi (2020). However, this may not be the complete picture of the reinsurance market. In particular, the following arguments could be considered as well:

- the reinsurer may benefit from diversification, because it has more risk-bearing capacities to pool multiple risks;
- asymmetric information, where the reinsurer may have less information than the insurer about the underlying distribution of the risk;
- capital constraints set by the regulator, that prevent the insurer from retaining the risk.

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Notes on contributor

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