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Approximate Bayesian inference based on INLA algorithm

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ABSTRACT

The integrated nested Laplace approximation (INLA) algorithm provides a computationally efficient approach for approximate Bayesian inference, overcoming the limitations of traditional Markov chain Monte Carlo (MCMC) methods. This paper reviews INLA algorithm and provides a systematic review of six key books that explore the theoretical foundations, practical implementations, and diverse applications of INLA. These six books cover spatial and spatio-temporal modelling, general Bayesian inference, SPDE-based spatial analysis, geospatial health data, regression modelling, and dynamic time series. In addition, these books highlight the versatility of INLA method in handling complex models while maintaining high computational efficiency. This paper begins with an introduction to the INLA method and algorithm, followed by a systematic review of six key publications in the field.

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Approximate Bayesian inference; INLA; computational efficiency; spatial; spatio-temporal

1. Introduction of INLA method

Bayesian inference, widely employed in statistical modelling and machine learning, combines prior distributions with likelihood functions via Bayes' theorem to derive posterior distributions for model fitting and prediction. In Bayesian inference, priors can take two forms: (1) subjective priors informed by historical data or domain expertize, or (2) objective priors constructed using formal rules such as Laplace's flat prior, Jeffreys prior, or reference priors. Bayesian inference is not only applicable to simple models with explicit common posterior distributions, but also to complex models with many parameters in the model or latent structure. Such complex models are usually difficult to explicitly represent the posterior distribution as they often involve complex high-dimensional integral calculations, for which the traditional numerical integration approximation and Monte Carlo integration are obviously unable to cope with the 'curse of dimensionality' problem. The amount of computation involved in the posterior inference increases exponentially with the increase of dimensionality of the parameters.

With the introduction and subsequent popularization of the Markov chain Monte Carlo (MCMC) algorithm in the 1980s, the challenge of solving complex integrals in Bayesian

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inference was effectively addressed by iteratively generating samples from the posterior distribution via a Markov chain. In parallel, to substantially reduce programming complexity, see influential early references, such as Hastings (1970), Geman and Geman (1984), Gelfand and Smith (1990) and Robert et al. (2004). Various Bayesian inference software tools based on probabilistic programming paradigms such as WinBUGS, OpenBUGS, JAGS, NIMBLE, and Stan, along with corresponding R packages, such as `R2WinBUGS`, `R2OpenBUGS`, `rjags`, `R2jags`, `runjags`, `nimble`, and `rstan`, as well as `Python` libraries like `PyMC3`, have been progressively developed and released. These tools have contributed to the emergence of a series of intelligent Bayesian inference engines primarily based on the BUGS language. These softwares and packages are now widely applied to analyze complex models with massive data size arising from diverse domains including sociology, ecology, environmental science, econometrics, finance and economics, biomedicine, epidemiology, and insurance. This widespread application, in turn, brings challenges in computation and creates new opportunities for further optimization and long-term advancement of MCMC algorithms. Consequently there has been a surge in accelerating and parallel MCMC algorithms in the last three decades; see a thorough review by Robert et al. (2018). However, these more efficient algorithms introduce tuning parameters which are usually not adaptive.

The MCMC algorithm is essentially an iterative algorithm of rejection-acceptance sampling, and the key is to indirectly extract Markov chains from the posterior distribution. Under certain regular conditions, it can be guaranteed that the iterative values from the Markov chain when reaching the stationary state can be used as a posterior sample for Monte Carlo integrals. This process is the origin of the acronym MCMC. Theoretically, given a sufficient number of iterations, the iterative values provided by the MCMC algorithm will eventually converge to our pre-specified target distribution, i.e., the posterior distribution. Bayesian inference based on MCMC algorithms, such as the typical Metropolis-Hastings algorithm and Gibbs sampling, can theoretically be regarded as an accurate posterior inference method. However, high precision usually suffers from long-run iterations to achieve stationarity, especially for those complex models with highly correlated high-dimensional parameters. In particular, the sampling efficiency of the MCMC algorithm depends on some of the following skills involved in the actual use.

(1) Selection of the Markov transition kernels: Poor transition kernels will cause the speed of the convergence to become very slow. The posterior samples may be strongly correlated, or the percentage of acceptance is very low.
(2) Convergence diagnosis: This usually requires human intervention, either through a convergence diagnostic diagram of multiple strands (e.g., sample trace plots, ergodic mean plots, autocorrelation plots), or by examining diagnostic indicators such as Markov chain errors and the Brooks-Gelman-Rubin (BGR) diagnostic statistic, commonly referred to as the potential scale reduction factor (PSRF).
(3) Selection of valid samples: In order to ensure the accuracy of the inference, the convergent Markov model still needs to discard a part of the initial iterative values (called burn-in) and select a part of the iterative values (called thinning) at the final posterior sample, which will directly affect the accuracy of posterior inference.

Therefore, theoretically perfect MCMC algorithms still face the 'curse of dimensionality' problem in practical use, which manifests as a scalability issue in computation and, to a large extent, hinders the implementation of Bayesian inference for complex models.

In the past twenty years, approximate Bayesian inference has emerged quietly, at nearly the same time as accelerating MCMC. Approximate Bayesian computation (ABC), variational Bayes (VB) and INLA are the three types of computation methods rooted in Bayesian statistics. ABC methods date back to the 1980s (Rubin, 1984). As it bypasses the evaluation of the likelihood function, it rapidly gained popularity over the last twenty years and in particular for the analysis of complex problems arising in biological sciences. See Sunnåker et al. (2013) for a detailed introduction about ABC. VB, an optimization-based technique for approximate Bayesian inference (Attias, 1999), makes use of the mean field approximation, making a factorized approximation to the true posterior. VB is computationally efficient and can be applied to a large class of probabilistic models in Bayesian statistics and machine learning (Peterson, 1987; Winn & Bishop, 2005). INLA is a computationally efficient method for approximate Bayesian inference in latent Gaussian models, using nested Laplace approximations to estimate posterior marginal distributions without relying on Markov chain Monte Carlo (MCMC) simulations. It is particularly well-suited for large-scale spatial, temporal, and spatio-temporal models. See Rue et al. (2017) and Van Niekerk et al. (2023) for the development and diverse applications of INLA from two review papers in two periods (before and after 2016) in history since the advent of INLA.

2. INLA algorithm

This paper serves as reviews of six books on INLA. Thus we simply describe the approximate Bayesian method first. Proposed by Rue et al. (2009), INLA aims to provide a fast and accurate approximate Bayesian computational method. A latent Gaussian model (LGM) is essentially a hierarchical Bayesian model that consists of a likelihood function with linear predictors, a latent Gaussian random field (LGRF), and a prior distribution for a vector of hyperparameters, which can be expressed mathematically as follows:

$$\begin{aligned} \boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\theta}_1 &\sim \prod_i p\left(y_i \mid \eta_i(\boldsymbol{x}), \boldsymbol{\theta}_1\right), \\ \boldsymbol{x} \mid \boldsymbol{\theta}_2 &\sim N\left(\mathbf{0}, \mathbf{Q}^{-1}\left(\boldsymbol{\theta}_2\right)\right), \\ \boldsymbol{\theta} &= (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) \sim \pi(\boldsymbol{\theta}), \end{aligned} \tag{1}$$

where the linear predictor η_i is composed of the latent variable vectors $\boldsymbol{x}$ and other covariates, $\boldsymbol{\theta}$ is the vector of hyperparameters in the LGM, $\pi(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, and $\mathbf{Q}(\boldsymbol{\theta}_2)$ is the precision matrix. The likelihood function $p(\cdot|\cdot,\cdot)$ in the model has no restrictions, and the linear predictor η_i can include linear fixed effects and linear or nonlinear random effects, which can also smooth effects, spatial effects, or temporal effects. It can be seen that LGM can include many complex models, such as well-known generalized linear models (GLM), generalized additive models (GAM), time series models, spatial models and measurement error models.

LGRF is also known as Gaussian Markov Random Field (GMRF) (Rue & Held, 2005), and its latent effect $\boldsymbol{x}$ satisfies Markov property and normality, where Markov property guarantees the following: (1) the conditional independence between the latent variables, i.e., $x_i \perp x_j|\boldsymbol{x}_{-ij}$; (2) for $i \neq j$, $x_i \perp x_j|\boldsymbol{x}_{-ij}$ if and only if $\mathbf{Q}_{ij}(\boldsymbol{\theta}_2) = 0$, i.e., the precision matrix is sparse. In this way, although $\boldsymbol{x}$ is usually high-dimensional, the sparsity of the precision matrix and the low-dimensional characteristics of the hyperparameter vector $\boldsymbol{\theta}$ ensure that the parameters to be estimated in this model can be greatly reduced, which is the key to the rapid implementation of Bayesian calculations by the INLA method.

Based on the above theory, Rue et al. (2009) developed the INLA algorithm to calculate the posterior distribution of the hyperparameter vector $\boldsymbol{\theta}$, $\pi(\boldsymbol{\theta}|\boldsymbol{y})$, and the marginal posterior distribution of the latent effect x_j. Here the 'Laplace approximation' is applied to the conditional posterior distribution of $\boldsymbol{x}$, and the 'nested' is applied to the numerical integral approximation. In order to facilitate the popularization and use of the algorithm, Rue et al. (2009) developed the `R` package `INLA`, also called `R-INLA` based on the `C` library of the same name, GMRF, which has been quite stable and widely used for more than ten years. In addition, in order to implement Bayesian analysis of spatial data on geographic regions, Lindgren et al. (2011) pointed out that Gaussian continuous space processes with Matérn covariance structure can be used as a solution to stochastic partial differential equation (SPDE) for LGRF on approximate continuous space. Moreover, they developed an algorithm for this LGRF based on the finite element method and created the R package `inlabru`, which extends the `R-INLA` package and also enables geographic mapping. Finally, through these two packages, it becomes feasible to implement spatio-temporal statistical modelling that integrates both temporal and spatial processes.

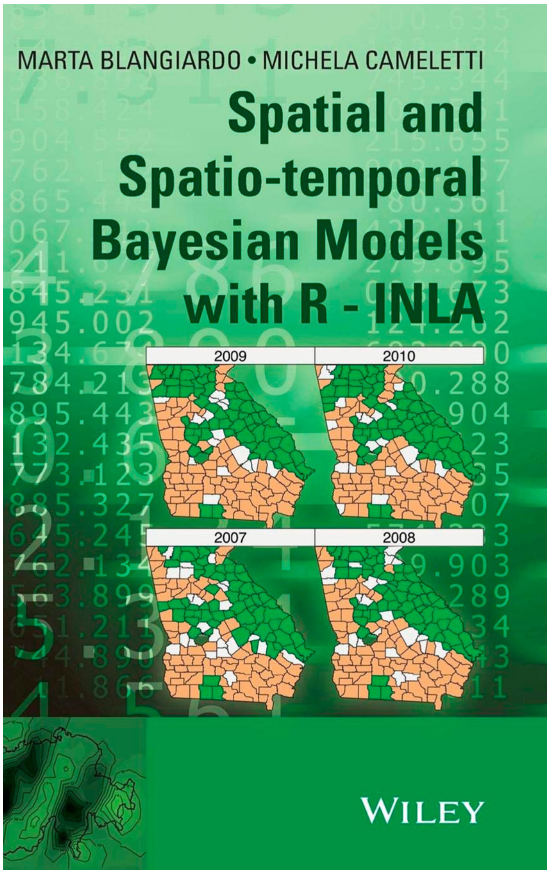
3. Review of six books

Over the past decade, the INLA algorithm and the `R-INLA` software package have gained widespread adoption, as evidenced by their extensive presentation in numerous research papers, case studies, and inclusion in high-quality, peer-reviewed publications. In the past two years, we have systematically studied the core chapters of the INLA-related literature through seminar-style sessions, reproduced a substantial number of illustrative examples, and thereby empirically validated the accuracy and computational efficiency of the INLA algorithm, as well as the user-friendliness of the `R-INLA` package. Our study focussed on the following six key publications:

(1) Blangiardo, M., and Cameletti, M. (2015). *Spatial and Spatio-temporal Bayesian Models with R-INLA*. John Wiley & Sons.
(2) Gómez-Rubio, V. (2020). *Bayesian Inference with INLA*. Chapman & Hall/CRC.
(3) Krainski, E., Gómez-Rubio, V. Bakka, H., Lenzi, A., Castro-Camilo, D., Simpson, D., Lindgren, F., and Rue, H. (2018). *Advanced Spatial Modelling with Stochastic Partial Differential Equations Using R and INLA*. Chapman & Hall/CRC.
(4) Moraga, P. (2019). *Geospatial Health Data: Modelling and Visualization with R-INLA and Shiny*. Chapman & Hall/CRC.
(5) Wang X., Yue, Y. R., and Faraway, J. J. (2018). *Bayesian Regression Modelling with INLA*. Chapman & Hall/CRC.
(6) Ravishanker, N., Raman, B., and Soyer, R. (2022). *Dynamic Time Series Models using R-INLA: An Applied Perspective*. Chapman & Hall/CRC.

We now provide an overview of the key characteristics and contributions of each publication individually.

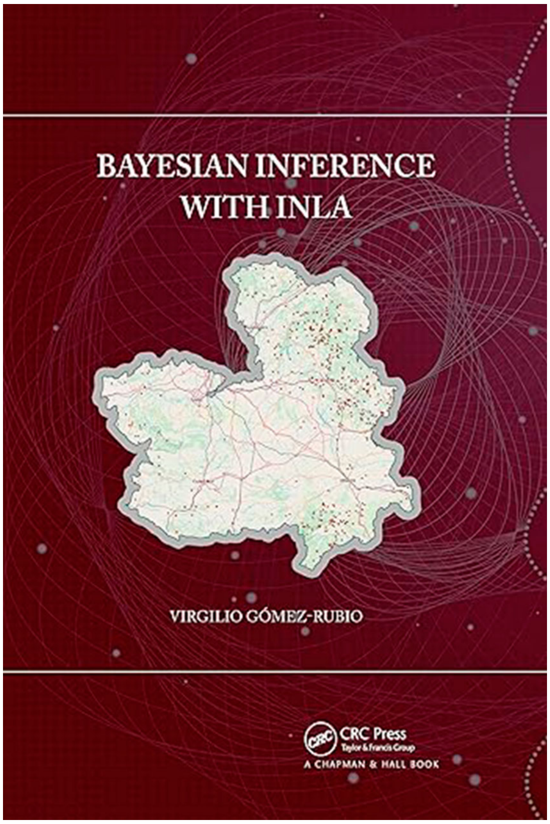
3.1. Spatial and spatio-temporal Bayesian models with R-INLA



This book, written by Marta Blangiardo and Michela Cameletti (2015), provides a comprehensive introduction to Bayesian thinking and theoretical aspects of the Bayesian approach. It focuses on the spatial and spatio-temporal models used within the Bayesian framework, and a series of practical examples that enable readers to connect statistical theory with real data problems.

The book is suitable for beginners interested in spatial and spatio-temporal data analysis, but with only elementary knowledge in statistics, lacking skills in R programming and just starting to learn Bayesian modelling. The first four chapters introduce the fourteen datasets used in the book, R basics, Bayesian methods and Bayesian computing, followed in the next chapters by detailed introduction on Bayesian modelling of linear regression, nonlinear regression, generalized linear models and hierarchical models, which are fundamental for Bayesian modelling of spatial and spatio-temporal model in the last three chapters. Besides, binomial and Poisson zero-inflated models and an advanced bivariate model are also introduced for special features in the real data sets. With focus on the spatial and spatio-temporal Bayesin models with `R-INLA`, the wealth of examples are analyzed step-by-step accompanied by detailed R scripts and explanation from the output of the codes.

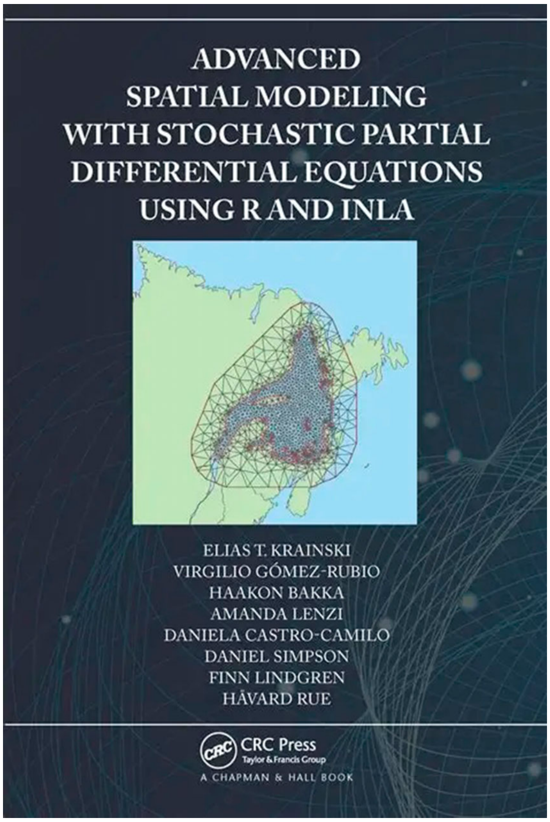
3.2. Bayesian inference with INLA



The statisticians will gain a comprehensive understanding of Bayesian inference and its applications in `R-INLA` through this book by Professor Virgilio Gómez-Rubio (2020), who received the SEIO-FBBVA Awards 2022 in data science and big data because of this widely adopted reference book in the field of Bayesian inference. The primary objective of this book is to introduce the INLA method and the associated `R-INLA` package. The book systematically achieves its dual purpose: introducing the INLA methodology and demonstrating its implementation in R. The online version of this book is available at https://becarioprecario.bitbucket.io/inla-gitbook, and the R package `brinla` is available at https://github.com/julianfaraway/brinla.

The book begins with a concise introduction to Bayesian inference, providing context for the INLA approach. Following this, it offers an exhaustive explanation of the INLA method, accompanied by two illustrative examples demonstrating the use of the INLA package within the R statistical environment. Subsequent chapters develop various widely used models, including mixed-effects model, multilevel model, spatial model, temporal model, smoothing techniques, survival model and others. Additionally, the book covers the specification of prior distributions in INLA, advanced features, the implementation of new latent models, handling missing values and imputation, as well as mixture models.

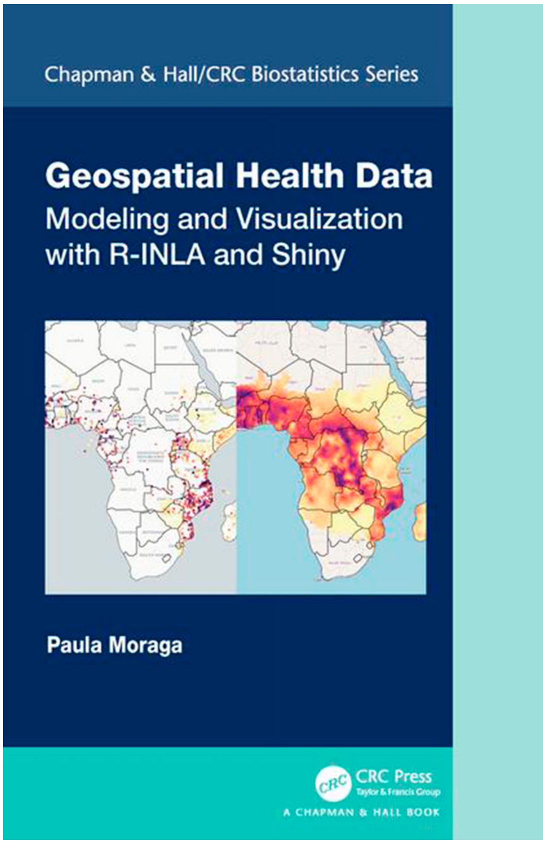
3.3. Advanced spatial modelling with stochastic partial differential equations using R and INLA



Written by Elias Krainski, Virgilio Gómez-Rubio, Haakon Bakka, Amanda Lenzi, Daniela Castro-Camilo, Daniel Simpson, Finn Lindgren, and Håvard Rue (2018), this book introduces the fundamental theory and application of spatial modelling with SPDEs using `R-INLA`. It demonstrates how to fit models containing at least one effect specified with SPDEs using the R package `INLA` for statistical computing. This method describes an approximation to continuous spatial models with the Matérn covariance that is based on the solution to an SPDE. An SPDE-based model is used to define random effects over continuous domains in one or two dimensions. This book focuses on SPDE models with INLA without covering the basics of Bayesian inference or spatial analysis.

One of the standout features of the book is its application of the SPDE methods. It introduces key concepts like Gaussian random fields, the SPDE approach, and mesh construction before diving into practical examples, including non-Gaussian data, point pattern analysis, and space-time modelling. The inclusion of diverse case studies such as precipitation modelling in Paraná and noise data analysis in Albacete demonstrates the versatility of SPDE-INLA methods. Additionally, the book provides online resources, including R code and datasets, ensuring reproducibility at https://becarioprecario.bitbucket.io/spde-gitbook.

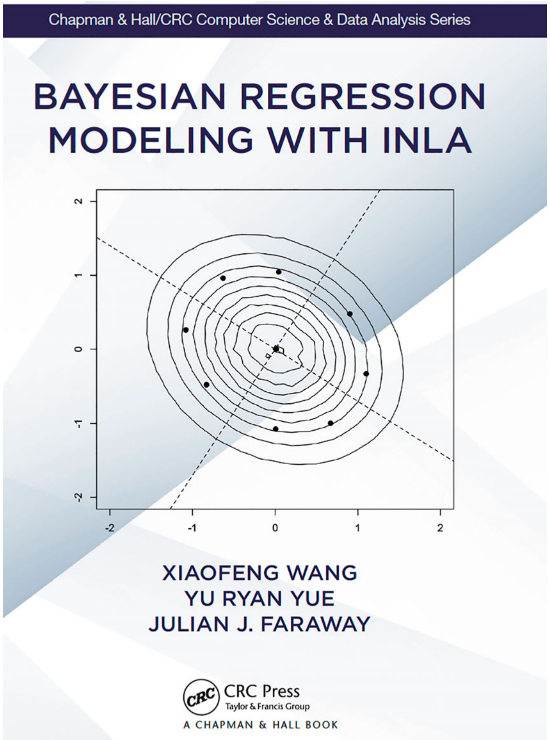
3.4. Geospatial health data: modelling and visualization with R-INLA and shiny



This book, written by Paula Moraga (2019), provides an exceptional guide to analyzing and visualizing geospatial health data using advanced Bayesian methods in R. It is primarily aimed at epidemiologists, biostatisticians, public health specialists, and professionals of government agencies working with georeferenced health data. Readers will gain hands-on experience using INLA to quantify disease risk, identify patterns, and communicate findings effectively. The inclusion of `Shiny` applications and interactive dashboards ensures that users can transform complex analyses into accessible visual tools for policymakers and stakeholders.

This book presents concrete implementations of Bayesian spatial and spatio-temporal models using `R-INLA`. It shows how to develop Bayesian hierarchical models and apply computational approaches such as INLA and SPDE to analyze data collected in areas and at particular locations by disease registries, national and regional institutes of statistics, and other organizations. These approaches allow one to quantify the disease burden, understand geographic and temporal patterns, identify risk factors, and measure social inequality. It includes the spatial modelling of areal lip cancer in Scotland, spatio-temporal modelling of areal lung cancer in Ohio, spatial modelling of geostatistical malaria in the Gambia, spatio-temporal modelling of geostatistical air pollution in Spain as examples. Notably, this book provides comprehensive coverage of specialized tools including `rmarkdown`, `flexdashboard`, `Shiny`, and `SpatialEpiApp` for advanced modelling and geospatial visualization of health data. The online version of the book based on R package `bookdown` is available at https://www.paulamoraga.com/book-geospatial/index.html.

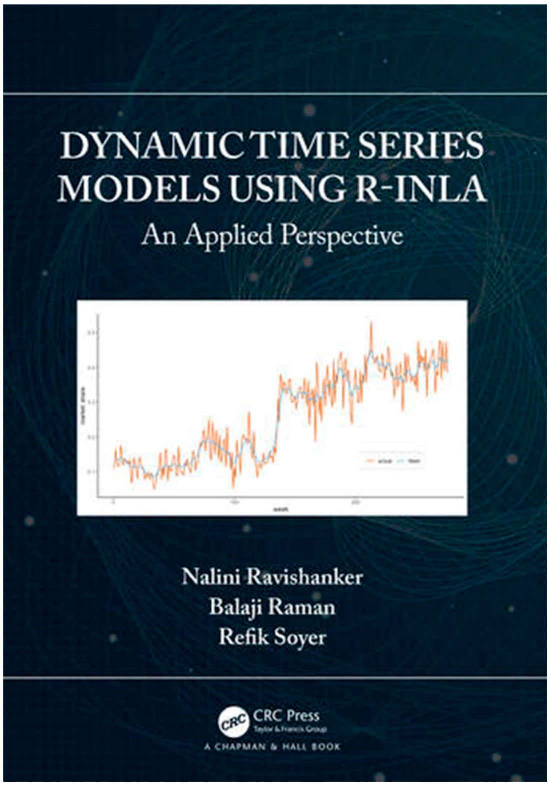
3.5. Bayesian regression modelling with INLA



This book was written by Xiaofeng Wang, Yu Ryan Yue and Julian J. Faraway (2018) and offers a groundbreaking approach to Bayesian modelling through INLA, presenting a faster and more accessible alternative to traditional MCMC methods. The authors expertly bridge theory and practice, making advanced Bayesian techniques feasible for real-world applications. The R scripts for all the examples in each chapter are available on http://julianfaraway.github.io/brinla/. For the sake of usage, the author developed a separate R package `brinla` which contains data and functions to support the book.

The book begins with a clear introduction to Bayesian inference and the computational advantages of INLA, followed by detailed coverage of its implementation in R. What sets this book apart is its comprehensive treatment of modern regression models from linear and generalized linear models to mixed-effects, spatio-temporal, and survival analysis frameworks all demonstrated through INLA efficient approximations. This book is invaluable for statisticians and data scientists seeking to leverage Bayesian methods without computational bottlenecks, making it a must-have for applied researchers and graduate students alike.

3.6. Dynamic time series models using R-INLA: an applied perspective



This book was written by Nalini Ravishanker, Balaji Raman and Refik Soyer (2022) and is a comprehensive and practical guide for applied statisticians and data scientists working with time series data. It introduces the INLA method as a fast and efficient alternative to traditional Bayesian inference techniques like MCMC. Readers will gain a deep understanding of how to implement dynamic Bayesian models using the `R-INLA` package, with a focus on real-world applications. The authors provide clear explanations, step-by-step instructions, and numerous examples, making complex concepts accessible. The online version of this book is available at https://ramanbala.github.io/dynamic-time-series-models-R-INLA, and the R codes and data sets are available at https://github.com/ramanbala/dynamic-time-series-models-R-INLA.

This book is particularly valuable for those seeking to analyze time series data with speed and accuracy, offering practical insights into model selection, forecasting, and hyperparameter estimation. A key strength of the book lies in its systematic treatment of various time series models, including univariate, time series regression models, hierarchical dynamic models for panel time series, models for non-Gaussian continuous response, categorical time series, count time series, stochastic volatility, multivariate Gaussian dynamic model and hierarchical multivariate time series. Examples include Musa software engineering example, monthly average cost of nightly hotel stay, ridesourcing in NYC, volatility index time series, weekly shopping trips for multiple households, daily bike rentals in Washington D.C, IBM stock returns, and monthly TNC usage in NYC taxi zones. With its blend of theory and hands-on R codes, this book serves as both a learning resource and a reference for practitioners leveraging `R-INLA` for time series analysis.

4. Conclusion

The aforementioned six books offer a comprehensive and detailed exposition of the INLA algorithm, encompassing its theoretical foundations, practical applications, and corresponding code implementations with high efficient R package `R-INLA`. Furthermore, each book is distinguished by its focus on specific data types and analytical contexts. The material in these six books would require careful reading to understand the analytical details and how these methods can be applied to research.

Though the INLA approach and package can help us perform fast and accurate approximate Bayesian inference for a wide range of large models, its application is restricted to LGMs and its computational efficiency can decrease with a high number of hyperparameters. However, these limitations are offset by its profound utility. For the wide array of models within its scope, INLA remains an exceptionally efficient and robust alternative to simulation-based methods, greatly expanding access to sophisticated Bayesian analysis. Recently, there remain four notable extensions of INLA that draw our attention with corresponding new R packages which can be combined with the standard INLA package to address some of these limitations and further enhance its capabilities.

(1) `inlabru`: Bachl et al. (2019) extended the GAM-like model class to more general non-linear predictor expressions, and implemented a log Gaussian Cox process likelihood for modelling univariate and spatial point processes based on ecological survey data.
(2) `INLA`$^+$: Abdul-Fattah et al. (2023) developed an approximated Bayesian approach for spatial models with non-sparse precision or covariance matrices. It scales better compared to the standard INLA, and computational power by multiprocessors in shared and distributed memory architectures.
(3) `rSPDE` (with INLA): Bolin et al. (2024) provided a computationally efficient Bayesian approach to approximate the fractional power in modelling large spatial datasets with SPDE approach. `rSPDE` is an R package developed by David Bolin, used for computing rational approximations of fractional SPDEs, which will result in a numerically stable GMRF approximation combined with the INLA method for fast Bayesian inference.
(4) `INLAjoint`: Alvares et al. (2024) presented how various Bayesian survival models can be fitted using the INLA package in a clear, legible, and comprehensible manner using the `INLA` and `INLAjoint` R packages, including accelerated failure time, proportional hazards, mixture cure, competing risks, multi-state, frailty, and joint models of longitudinal and survival data.

Disclosure statement

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