

Statistical Theory and Related Fields

ISSN: 2475-4269 (Print) 2475-4277 (Online) Journal homepage: www.tandfonline.com/journals/tstf20

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To cite this article: Liangliang Zhuang , Nan Liu , Di Zhu & Ancha Xu (26 Nov 2026): Multi-phase degradation modelling: a review of model structures, statistical inference, and decision support, Statistical Theory and Related Fields, DOI: 10.1080/24754269.2026.2762676

To link to this article: https://doi.org/10.1080/24754269.2026.2762676

Published online: 26 Nov 2026.

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STATISTICAL THEORY AND RELATED FIELDS
https://doi.org/10.1080/24754269.2026.2762676

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Multi-phase degradation modelling: a review of model structures, statistical inference, and decision support

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ABSTRACT

Degradation analysis provides an important basis for reliability assessment, remaining useful life (RUL) prediction, and maintenance decision-making. However, many engineering systems exhibit distinct degradation phases due to changes in operating conditions or underlying degradation mechanisms. Ignoring such phase transitions may lead to inaccurate lifetime predictions and inappropriate maintenance decisions. This paper presents a systematic review of multi-phase degradation analysis. We first introduce the basic concepts and characteristics of multi-phase degradation, and then summarize major modelling approaches, including general path models and stochastic process models. We further discuss inference and identification issues, including parameter estimation and change-point detection, as well as downstream applications in RUL prediction and maintenance optimization. Finally, open challenges and future directions are discussed in terms of mechanism-informed modelling, uncertainty quantification, generalizable prediction, and artificial-intelligence-enabled phase-aware degradation analysis.

ARTICLE HISTORY
Received 14 July 2026
Revised 14 October 2026
Accepted 7 November 2026

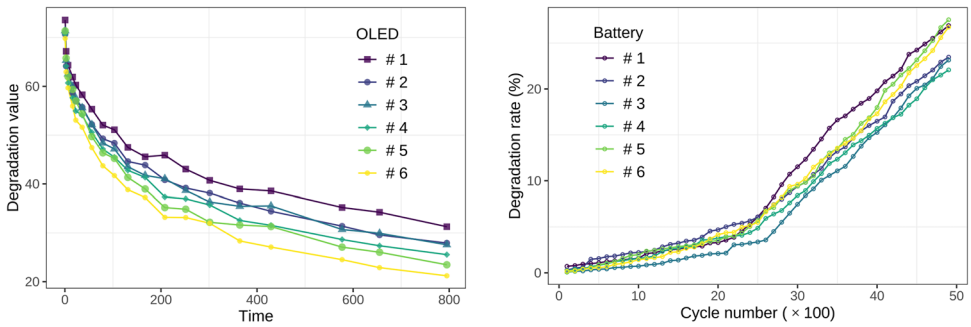
KEYWORDS
Multi-phase degradation; change-point; degradation modelling; remaining useful life prediction; maintenance decision-making

1. Introduction

Degradation data obtained from online monitoring provide an important basis for reliability assessment, RUL prediction, and maintenance planning, as they describe the temporal evolution of system performance through signals such as vibration, temperature, voltage, capacity, and luminance. In many engineering systems, however, degradation trajectories may change their evolution patterns as damage accumulates, operating conditions vary, or failure mechanisms shift, leading to two-phase or multi-phase behaviour. Figure 1 shows two representative examples: organic light-emitting diode (OLED) luminance degradation often presents a rapid initial decline followed by slower aging (Wang & Bae, 2026), while lithium-ion battery capacity degradation may shift from slow decay to accelerated degradation (L. Zhuang et al., 2024). Similar phase-switching behaviour is also observed in cutting tool wear (X. Zhang et al., 2023) and bearing fatigue degradation (J. Wang, Ma, & Zhang, 2025). If such

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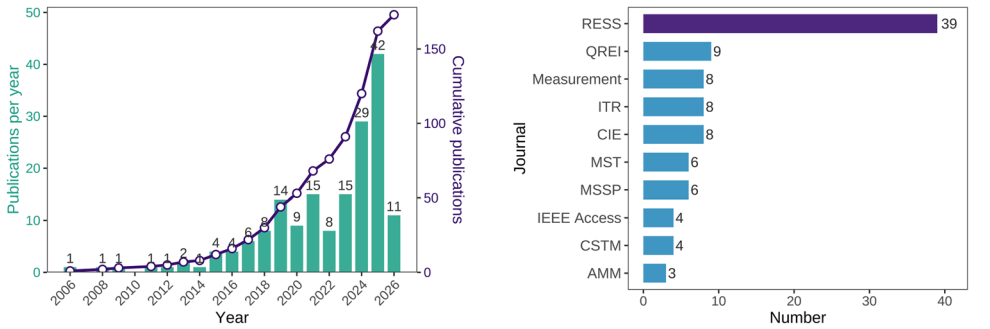
(a) OLED data (b) lithium-ion battery data

Figure 1. Representative multi-phase degradation datasets: (a) OLED data (Wang & Bae, 2026); (b) Lithium-ion battery data (L. Zhuang et al., 2024).

phase transitions are ignored, degradation trajectories and failure-time distributions may be misspecified, leading to biased RUL prediction and suboptimal maintenance decisions (Ye & Xie, 2015). These issues motivate multi-phase degradation analysis, which aims to identify phase boundaries, characterize phase-specific degradation dynamics, and incorporate phase information into lifetime prediction and maintenance optimization.

Several reviews have examined multi-phase time-series analysis and related structural-change problems. General reviews mainly summarize statistical methods and algorithms for change-point detection in sequential data (Gupta et al., 2024; Niu et al., 2016; Truong et al., 2020). For example, Niu et al. (2016) reviewed multiple change-point detection from a statistical perspective, while Truong et al. (2020) organized offline algorithms according to cost functions and search strategies. Other reviews have focussed on application-specific change-point problems, such as control chart postsignal diagnostics (Amiri & Allahyari, 2012) and modern data analysis applications (J. Li, Wang et al., 2025). These studies provide useful foundations for analyzing structural changes in time-series data. However, in degradation analysis, phase transitions are often linked to degradation mechanisms and operating conditions, rather than being only statistical shifts in data patterns. Such transitions may change the degradation path and failure-time distribution, thereby affecting parameter inference, RUL prediction, and maintenance decision-making. Existing studies on multi-phase degradation have been developed across diverse modelling frameworks and application domains, but a systematic review that connects degradation modelling, parameter inference, RUL prediction, and maintenance decision-making is still lacking.

To address this gap, we collected relevant studies from Web of Science using keyword combinations related to degradation, two-phase, multi-phase, multi-stage, and change-point. Backward and forward citation tracking of representative papers was further conducted to identify additional relevant studies. After screening, 173 papers published up to April 2026 were retained for review. Figure 2 summarizes the publication trend and journal distribution of the reviewed literature, showing a general upward trend with a marked increase after 2023, with *Reliability Engineering & System Safety* as the most frequent publication venue. Based on these studies, this review provides a structured overview of multi-phase degradation analysis. We first clarify the basic concepts of multi-phase degradation, focussing on phase structures and transition behaviour. We then review major modelling frameworks, parameter



(a) Publication trend (b) Main journals

Figure 2. Overview of the reviewed literature: (a) publication trend and (b) journal distribution. The publication count for 2026 includes records available up to April 2026 only.

inference, and change-point detection methods. Applications in RUL prediction and maintenance decision-making are further discussed, followed by future research directions related to mechanism-informed modelling, uncertainty quantification, prediction generalization, and intelligent decision-making.

The remainder of this paper is organized as follows. Section 2 introduces the basic concepts of multi-phase degradation. Sections 3 and 4 review modelling methods, parameter estimation, and change-point detection. Section 5 discusses RUL prediction and maintenance decision-making. Section 6 concludes the paper.

2. Concepts of multi-phase degradation

Let $Y(t)$ denote the observed degradation level of a system at time $t \in [0, T]$, where T is the end of observation. A simplified representation of a single-phase degradation process is

$$Y(t) = \mathcal{G}(t; \boldsymbol{\theta}) + \varepsilon(t), \tag{1}$$

where $\mathcal{G}(t; \boldsymbol{\theta})$ denotes the underlying degradation trajectory, $\boldsymbol{\theta}$ is the parameter vector governing the degradation dynamics, and $\varepsilon(t)$ represents measurement error. In this setting, the degradation process is represented by a single evolution pattern with the same parameter structure assumed throughout the observation period (Meeker et al., 2022). A multi-phase degradation process allows the degradation dynamics to vary across different time intervals (B. Zhuang et al., 2026). Let $0 = \tau_0 < \tau_1 < \cdots < \tau_{K-1} < \tau_K = T$ define a partition of the observation interval, where K is the number of phases and $\tau_1, \ldots, \tau_{K-1}$ denote phase boundaries, also referred to as change points in this review. The observed degradation process can be expressed as

$$Y(t) = \mathcal{M}_k\big(t; \boldsymbol{\theta}_k\big) + \varepsilon(t), \quad t \in (\tau_{k-1}, \tau_k],\ k = 1, \ldots, K, \tag{2}$$

where $\mathcal{M}_k(\cdot)$ denotes the phase-specific degradation structure, and $\boldsymbol{\theta}_k$ is the parameter vector associated with phase k. Depending on the model, $\mathcal{M}_k(\cdot)$ may be specified as a deterministic path function, a stochastic process, a state-space model, or another phase-specific formulation. This piecewise formulation allows different degradation phases to be described by different evolution patterns and parameter structures.

Besides phase partitioning, another important characteristic of multi-phase degradation is how adjacent phases are connected at the change point. As illustrated in Figure 3, phase

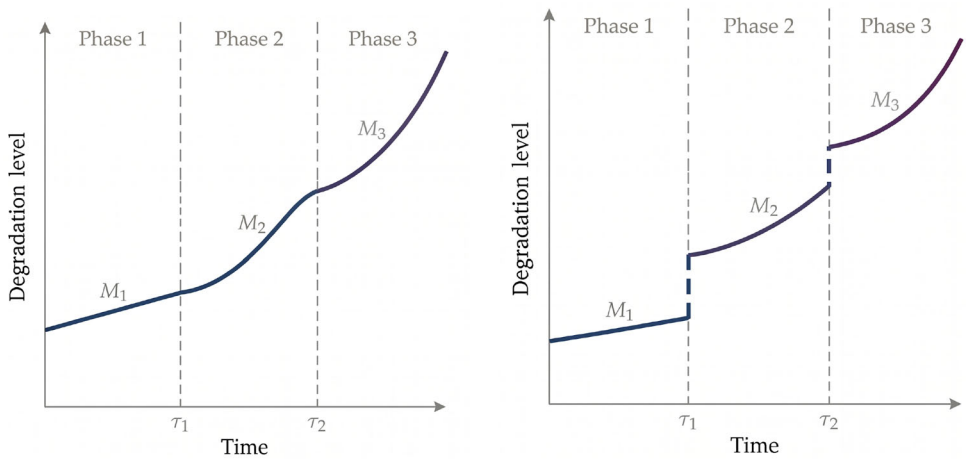


Figure 3. Typical phase-transition patterns in multi-phase degradation: (a) continuous transition; (b) jump transition.

transitions can be represented as either continuous or jump transitions. In a continuous transition, the degradation level is preserved at the change point, whereas in a jump transition it changes abruptly. Within the general formulation in Equation (2), these transition forms can be specified through the connection between adjacent phase-specific structures. Both transition forms have been adopted in the literature depending on the underlying degradation mechanism and modelling assumptions. Building on the above concepts, the following section reviews modelling frameworks for multi-phase degradation.

3. Multi-phase degradation models

Figure 4 summarizes the modelling taxonomy adopted in this review, including general path models, classical stochastic process models, and extended modelling frameworks. The following subsections introduce these modelling approaches and their representative developments.

3.1. General path models

General path models represent one of the earliest and most widely used approaches to degradation modelling. Under the general formulation in Equation (2), they can be regarded as a special case in which $\mathcal{M}_k(\cdot)$ is represented by a deterministic path function. These models describe the average degradation trajectory through explicit path functions. Specifically, for the kth phase, the observed degradation process can be written as

$$Y(t) = g_k\left(t - \tau_{k-1}; \boldsymbol{\beta}_k, \mathbf{x}\right) + \varepsilon(t), \quad t \in (\tau_{k-1}, \tau_k],\ k = 1, \ldots, K,$$

where $g_k(\cdot)$ is the phase-specific path function, $\boldsymbol{\beta}_k$ denotes the corresponding vector of path parameters, $\mathbf{x}$ represents covariates, and $\varepsilon(t)$ is the measurement error. Continuity constraints can be imposed at phase boundaries, while jump transitions allow discontinuities between adjacent phase-specific paths. Given a phase partition, $g_k(\cdot)$ can be specified according to physical mechanisms or empirical patterns. Linear forms are commonly used to describe approximately constant degradation rates, whereas nonlinear forms are adopted

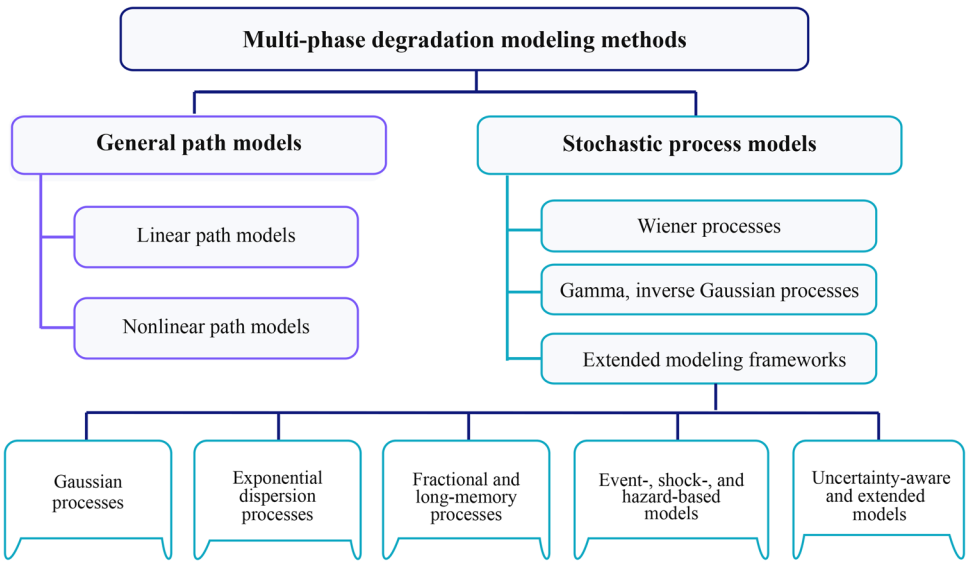


Figure 4. Taxonomy of multi-phase degradation modelling methods.

for more complex degradation behaviours. Covariates and random effects can also be incorporated to account for operating conditions and unit-to-unit heterogeneity.

Early studies on general path models mainly focussed on two-phase degradation using piecewise or change-point regression. A representative example is Bae and Kvam (2006), who proposed a two-phase change-point regression model with random coefficients for incomplete burn-in of display devices, dividing degradation into an initial rapid-decline phase and a subsequent stable-degradation phase. Later, Chen and Tsui (2013) extended the two-phase linear regression framework for condition monitoring and RUL prediction by incorporating empirical Bayes and online Bayesian updating to account for unit heterogeneity. Subsequent studies further introduced hierarchical structures and more flexible path functions to improve model flexibility and parameter estimation (Bae, Yuan, & Kim, 2016; Bae, Yuan, Ning et al., 2015; Yuan et al., 2016).

More recent work has extended this line of research toward richer phase structures and joint degradation–lifetime modelling. Representative developments include exact Bayesian inference for multiple change-point regression with online updating (Y. Wen, Wu, Zhou et al., 2019), mixed-effects regression models integrating condition-monitoring and failure-time information for unit-level RUL prediction (Son et al., 2015), and joint modelling of degradation paths and time-to-event data (Brumm et al., 2025). General path models provide a simple and interpretable description of phase-wise degradation trajectories. However, their deterministic nature limits the characterization of within-phase stochastic variability and uncertainty, motivating the development of stochastic process models.

3.2. Stochastic process models

3.2.1. Wiener processes

Wiener processes are the most widely used stochastic process models for multi-phase degradation. Under the general formulation in Equation (2), they characterize both the average degradation trend and within-phase stochastic variability through phase-dependent drift and

Table 1. Representative extensions of phase-wise Wiener degradation modelling.

Category	Features	Representative studies
Unit heterogeneity	Random effects for phase-specific model parameters	H. Wang, Ma et al. (2020), Cui et al. (2024), Sun, Li et al. (2024), and B. Zhuang et al. (2026)
Measurement error	Separation of latent degradation and observation processes	P. Wang, Tang, Bae, & Xu (2018), Guan, Wei et al. (2022), Lin and Chai (2023), and Yao and Zhu (2026)
Nonlinear degradation	Phase-specific nonlinear drift functions or adaptive functions	J. Lin, Liao et al. (2021), Hu et al. (2023), Z. Wang, Zhang et al. (2020), Z. Wang, Jiang et al. (2025), and Z. Xu et al. (2025)
Jump and shock effects	Jump terms, fuzzy change points, or shock-degradation coupling	Kong et al. (2017), Liu, Dang et al. (2022), and H. Gao et al. (2020)
Multivariate degradation	Bivariate and multivariate dependence structures	B. Yan et al. (2023), L. J. Sun, Liang et al. (2025), B. Wen et al. (2025), Sun, Li et al. (2024), and L. Fan et al. (2024)

diffusion terms. A phase-specific Wiener model can be expressed as

$$Y(t) = Y(\tau_{k-1}) + \mu_k \Lambda_k(t - \tau_{k-1}) + \sigma_k B_k(t - \tau_{k-1}), \quad t \in (\tau_{k-1}, \tau_k],\ k = 1, \ldots, K,$$

where μ_k and σ_k are the drift and diffusion parameters in phase k, respectively, $\Lambda_k(\cdot)$ is a phase-specific time-transformation function, and $B_k(\cdot)$ denotes a standard Brownian motion. For continuous transitions, $Y(\tau_{k-1}) = Y(\tau_{k-1}^-)$. Jump transitions can be accommodated by setting $Y(\tau_{k-1}) = Y(\tau_{k-1}^-) + J_{k-1}$, where J_{k-1} denotes the jump magnitude. When $\Lambda_k(s) = s$, the model reduces to a phase-wise linear Wiener process; more generally, power-law, exponential, or other nonlinear time transformations can be adopted to describe phase-specific nonlinear degradation.

Early studies mainly focussed on two-phase change-point Wiener models, where the degradation mechanism switches at an unknown time and phase differences are represented by piecewise drift or drift-diffusion structures. The single-change-point formulation was introduced into degradation modelling by Ng (2008), and was later extended to two-phase Wiener models with improved inference and RUL prediction methods (Prakash & Kaushik, 2020; P. Wang, Tang, Bae, & He, 2018; P. Wang, Tang, Bae, & Xu, 2018; W. A. Yan et al., 2017; J. X. Zhang et al., 2019). Subsequent studies have developed this basic framework along several directions. Table 1 summarizes representative extensions, which are discussed below.

Unit-to-unit heterogeneity is commonly represented by introducing random effects into phase-specific model parameters (Cui et al., 2024; H. Wang, Ma et al., 2020). Beyond capturing heterogeneity, these random-effect structures can also be used together with dependence models to describe correlations across indicators or degradation phases. For example, Sun, Li et al. (2024) developed a bivariate two-phase Wiener model that combines random effects with a copula structure to characterize cross-indicator dependence, whereas B. Zhuang et al. (2026) proposed a nonlinear phase-dependent Wiener model in which covariance between phase-specific parameters is used to describe dependence between adjacent phases. Measurement errors are commonly handled by separating the latent degradation process from noisy observations (Guan, Wei et al., 2022; P. Wang, Tang, Bae, & Xu, 2018). Representative examples include explicit observation-error modelling in nonlinear two-phase Wiener processes (Lin & Chai, 2023) and phase-dependent observation noise within state-space Wiener models (Yao & Zhu, 2026), thereby improving RUL prediction under noisy monitoring conditions.

Nonlinear degradation has been modelled mainly through phase-specific nonlinear drift functions and time-transformation functions, relaxing the assumption of piecewise linear

drift (J. Lin, Liao et al., 2021; Z. Wang, Zhang et al., 2020). Representative developments include generalized phase-dependent nonlinear formulations (Hu et al., 2023), adaptive drift functions (Z. Wang, Jiang et al., 2025), and smooth-transition two-phase formulations (Z. Xu et al., 2025). Abrupt state changes, such as the jump-transition case illustrated in Figure 3(b), present additional challenges for phase-wise Wiener modelling. Representative developments include explicit jump modelling at change points (Kong et al., 2017), fuzzy formulations for imprecisely located change points (Liu, Dang et al., 2022), and shock-triggered phase transitions with random degradation jumps (H. Gao et al., 2020). For systems monitored by multiple degradation indicators, multivariate Wiener models have been developed to characterize cross-indicator dependence. Typical formulations include correlation-driven bivariate Wiener models (B. Yan et al., 2023) and copula-based multivariate Wiener models (L. Fan et al., 2024; Sun, Li et al., 2024; L. J. Sun, Liang et al., 2025; B. Wen et al., 2025).

With increasing trajectory complexity, Wiener-process models have been extended from two-phase formulations to multiple-change-point and multi-phase structures, enabling the description of slow, accelerated, and rapid degradation stages (Feng et al., 2012; Y. Wen, Wu, Das et al., 2018; J. Zhang, He et al., 2017; J. Zhang, Si et al., 2019). Recent extensions further introduce nonlinear time transformations (Ji et al., 2025; W. Lin et al., 2024; Y. Wang, Liu et al., 2025), phase-dependent noise (H. Zheng et al., 2024), adaptive drift selection (S. Wang, Liu et al., 2024), and multi-source information correction or state-switching identification (Cao & Peng, 2023; Han et al., 2026; J. Yang et al., 2025; Z. Zhang et al., 2026), improving the flexibility and engineering applicability of multi-phase Wiener models.

Taken together, Wiener-process-based models provide a flexible framework for phase-wise degradation by jointly describing degradation trends and random fluctuations. However, their Gaussian-increment assumption may be less suitable for strictly monotone cumulative degradation, motivating the use of gamma and inverse Gaussian (IG) processes discussed next.

3.2.2. Gamma and inverse Gaussian processes

Gamma and IG processes are particularly suitable for monotone cumulative degradation because their increments are nonnegative. They have therefore been widely used for degradation phenomena such as wear, corrosion, capacity fading, and strength loss. In multi-phase settings, phase dependence is usually introduced by allowing the distributional or time-transformation parameters to vary across phases. The following phase-wise formulations assume no additional jumps at phase boundaries, with each phase starting from the degradation level reached at the end of the preceding phase.

For gamma processes, degradation increments over each phase follow a gamma distribution, naturally ensuring monotone degradation paths. A typical two-phase gamma process can be written as

$$Y(t) - Y(\tau_{k-1}) \sim \text{Gamma}\big(\alpha_k \Lambda_k (t - \tau_{k-1}), \beta_k\big), \quad t \in (\tau_{k-1}, \tau_k],$$

where α_k and β_k are the shape and scale parameters in phase k, respectively. Representative studies include a two-phase gamma process for LED luminous intensity degradation with Bayesian and stochastic expectation-maximization (EM) estimation (Ling et al., 2019), battery lifetime prediction using voltage features (C. P. Lin, Ling et al., 2021), and Bayesian-updated structural health monitoring (Prakash & Narasimhan, 2018). Related extensions use a gamma process and a Wiener process in different phases to construct change-point

Table 2. Representative extended or alternative modelling frameworks for multi-phase degradation.

Category	Features	Representative studies
Gaussian processes	Kernel-based degradation modelling and uncertainty quantification	Z. Chen et al. (2021), S. Chen et al. (2025), and Lim and Bae (2026)
Exponential dispersion processes	Mean-dispersion modelling and flexible increment distributions	Wang and Bae (2026) and Tian, Zhang et al. (2025)
Fractional and long-memory processes	Long-range dependence, memory effects, and non-Gaussian degradation fluctuations	H. Zhang, Zhou et al. (2019) and Duan et al. (2021)
Event-, shock-, and hazard-based models	Discrete-event dynamics, shock mechanisms, and phase-dependent failure-rate structures	P. Wang, Tang, & Xu (2019), Eryilmaz and Kan (2019), and Chadjiconstantinidis and Eryilmaz (2023)
Uncertainty-aware and extended models	Small-sample uncertainty, degradation propagation, and data-driven phase identification	Y. Wang, Kang et al. (2022), Cao et al. (2025), and H. Wang, Liao et al. (2022)

degradation models, with further analysis of robustness and reliability bias under model misspecification (Liu, Zou et al., 2020; X. Wang, Jiang et al., 2014).

IG processes also model monotone cumulative degradation, providing an alternative increment distribution to gamma processes. A typical phase-wise IG process can be expressed as

$$Y(t) - Y(\tau_{k-1}) \sim \mathrm{IG}\big(\mu_k \Lambda_k(t-\tau_{k-1}),\ \eta_k[\Lambda_k(t-\tau_{k-1})]^2\big), \quad t \in (\tau_{k-1}, \tau_k],$$

where μ_k and η_k are the phase-specific mean and shape parameters, respectively. Early studies focussed on two-phase IG degradation modelling and change-point identification (Duan & Wang, 2017). Subsequent work improved parameter estimation and maintenance-oriented lifetime analysis through reparameterized formulations and reliability-oriented inference (Gu et al., 2026; Liang et al., 2024; L. Zhuang et al., 2024). More recent developments further incorporate random effects for unit heterogeneity (Z. Z. Yan et al., 2025), random volatility for phase-dependent noise changes (Liang et al., 2026), and improved change-point detection based on modified information criteria or adjusted cumulative sum (CUSUM) statistics (M. Li et al., 2025; Qiao et al., 2025).

From a model-selection perspective, Wiener processes are suitable when degradation exhibits non-monotone fluctuations, whereas gamma and IG processes are preferable when nonnegative increments are physically required. The choice between gamma and IG processes depends on the distributional characteristics of degradation increments, particularly their variability and skewness, as well as goodness of fit to the observed data. However, most existing multi-phase gamma and IG models focus on two-phase degradation, and their extensions to more complex phase structures remain relatively limited.

3.2.3. Extended or alternative modelling frameworks

In addition to these classical stochastic processes, other modelling frameworks have also been explored for multi-phase degradation. These frameworks are grouped according to their main modelling features, including process structures, degradation mechanisms, and modelling strategies, and the resulting categories are not necessarily mutually exclusive. Table 2 provides an overview of representative studies, which are discussed below.

(1) *Gaussian process models.* Gaussian processes (GPs) use kernel functions to model degradation trajectories and quantify prediction uncertainty, making them suitable for nonlinear degradation, unknown mechanisms, and high-dimensional observations. Z. Chen et al. (2021) proposed a two-phase GP degradation model in which change points and covariance structures were jointly estimated using maximum likelihood, providing a probabilistic

framework for modelling degradation with phase transitions. Building on this framework, S. Chen et al. (2025) developed a stress-driven two-phase Gaussian process regression (GPR) model for accelerated degradation tests by incorporating stress-dependent piecewise mean functions and composite kernels to improve degradation modelling and cross-stress prediction. Extending GP models to more complex observations, Lim and Bae (2026) introduced a spatio-temporal Gaussian process with change points for image-based degradation data, enabling phase-specific modelling of spatially distributed degradation patterns. Overall, GP models provide flexible nonparametric representations and principled uncertainty quantification for multi-phase degradation analysis (W. Fan et al., 2025). However, their practical application still depends on appropriate kernel design, efficient hyperparameter estimation, and scalable inference for large datasets.

(2) *Exponential dispersion process models.* Exponential dispersion (ED) processes describe degradation increments through a mean-dispersion structure (Zhou & Xu, 2019), and Tweedie exponential dispersion (TED) processes further provide a unified distributional form for several classical degradation processes, thereby reducing the need to prespecify a particular degradation process (R. Chen et al., 2026; Tian, Bae et al., 2026). Wang and Bae (2026) proposed a Bayesian TED process with a change point for two-phase degradation data and derived approximate reliability and RUL inference for OLED luminance degradation. Tian, Zhang et al. (2025) further developed a two-phase TED process and used adaptive Bayesian synthetic likelihood to address cases with intractable likelihoods. These studies highlight the flexibility of ED/TED processes in modelling degradation increments. In practical applications, candidate models should be compared using information criteria or out-of-sample predictive performance, while goodness-of-fit diagnostics and predictive checks can help assess whether the selected model adequately captures the observed increment distributions and degradation trajectories.

(3) *Fractional and long-memory processes.* Fractional and long-memory processes relax the independent-increment assumption of classical stochastic processes, allowing degradation models to capture temporal dependence and, when combined with suitable switching structures, multiple degradation modes and non-Gaussian fluctuations (H. Zhang, Chen et al., 2021). Representative studies include fractional Brownian-motion models for multi-mode degradation and RUL inference (H. Zhang, Zhou et al., 2019), and fractional Lévy stable-motion models for long-memory and heavy-tailed degradation signals (Duan et al., 2021).

These models broaden stochastic-process-based degradation modelling by accommodating temporal dependence and non-Gaussian characteristics that are difficult to capture using classical Wiener processes.

(4) *Event-, shock-, and hazard-based models.* For systems dominated by discrete events, external shocks, or changes in failure-rate structures, phase switching can be modelled through point processes, shock models, and piecewise hazard-rate models. Typical studies include change-point compound Poisson processes with hierarchical Bayesian inference (P. Wang, Tang, & Xu, 2019), two-phase shock models with optimal replacement policies (Chadjiconstantinidis & Eryilmaz, 2023; Eryilmaz & Kan, 2019), generalized shock models with random or multiple shock-environment change points (X. Wang, Ning et al., 2023; Zhao, Qi et al., 2023), and shock models with phase-dependent self-healing effects (Zhao, Guo et al., 2018). Related studies also model phase transitions through failure-mechanism change-point analysis, piecewise lifetime distributions, or initiation-propagation degradation structures (Altun & Comert, 2016; J. Li, Tian et al., 2020; Zhou et al., 2025).

Nonhomogeneous Poisson process (NHPP) models provide an important framework for modelling phase-dependent failure intensity, particularly in software reliability growth and repairable-system reliability analysis. Multiple-change-point NHPP models have been developed to capture software reliability growth under imperfect debugging, fault dependence, and multi-version software evolution (Chatterjee et al., 2021; Huang & Lyu, 2011; Nageswari et al., 2025; Natarajan et al., 2025), and have also been extended to repairable systems and software reliability growth under changing operating environments (Bae, Mun et al., 2013; Dhaka & Nijhawan, 2024). Unlike continuous degradation-process models, NHPP-based methods characterize phase transitions through changes in event intensity or failure-occurrence processes, thereby providing a complementary perspective for modelling phase-dependent reliability evolution.

(5) *Uncertainty-aware and extended models.* Some studies further extend multi-phase degradation analysis by incorporating small-sample uncertainty, system-level degradation propagation, and data-driven phase identification. For small-sample settings, belief reliability and uncertain Liu processes have been introduced to model epistemic uncertainty arising from limited observations, thereby improving the robustness of parameter estimation (Y. Wang, Kang et al., 2022), while uncertain random processes have been proposed to jointly represent randomness, epistemic uncertainty, and measurement error (Cao et al., 2025). At the system level, degradation branching processes have been used to describe phase-wise degradation propagation across interacting components (H. Wang, Liao et al., 2022). In addition, time-series models, clustering methods, and neural networks have been applied to phase identification, degradation-pattern learning, and RUL prediction (Kundu et al., 2019; C. P. Lin, Cabrera et al., 2020; Singh et al., 2020). These studies extend multi-phase degradation analysis beyond conventional stochastic modelling by enabling uncertainty-aware inference, system-level degradation modelling, and data-driven phase characterization.

Overall, these extended and alternative modelling frameworks broaden multi-phase degradation analysis beyond classical processes. However, they also raise challenges in parameter estimation, model selection, computational efficiency, and engineering interpretation.

4. Parameter estimation and change-point detection

4.1. Parameter estimation methods

The model structures reviewed above lead to estimation problems involving phase-specific parameters and possibly unknown change-point numbers or locations. To address these estimation problems, Figure 5 summarizes three main classes of methods: likelihood-based estimation, Bayesian inference, and recursive updating, which are reviewed in the following subsections.

4.1.1. Likelihood-based estimation

Likelihood-based estimation is one of the most widely used approaches for multi-phase degradation models with explicit probabilistic structures. It estimates model parameters by maximizing the likelihood (or log-likelihood) of the observed degradation data. Let $\mathcal{Y}$ denote the observed degradation data and $\boldsymbol{\theta}$ the model parameters. The likelihood can be written as

$$L(\boldsymbol{\theta};\mathcal{Y}) = p(\mathcal{Y}\,|\,\boldsymbol{\theta}), \quad \ell(\boldsymbol{\theta};\mathcal{Y}) = \log L(\boldsymbol{\theta};\mathcal{Y}),$$

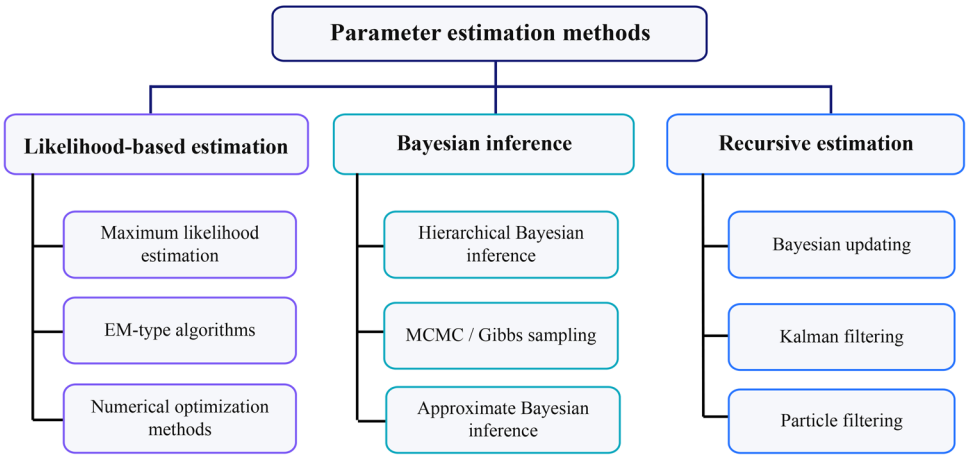


Figure 5. A taxonomy of parameter estimation methods in multi-phase degradation modelling.

and the estimate is obtained by $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}; \mathcal{Y})$. Here, $\hat{\boldsymbol{\theta}}$ denotes the maximum likelihood estimate (MLE) of the unknown parameter vector.

Existing studies have widely adopted MLE for multi-phase degradation models with tractable likelihood functions. Typical applications include two-phase Wiener, gamma, and IG processes (Duan & Wang, 2017; Gu et al., 2026; J. Lin, Shen et al., 2025; Ling et al., 2019). Likelihood-based estimation has also been applied to phase-wise failure-mechanism switching and hazard-rate models (Altun & Comert, 2016; J. Li, Tian et al., 2020), demonstrating its applicability to both stochastic degradation and reliability modelling. When the likelihood involves latent variables, incomplete observations, or hidden states, EM-type algorithms provide a natural extension of MLE. Early studies applied EM to estimate latent change points in degradation models (Ng, 2008), followed by stochastic EM for two-phase gamma processes with unknown change times (Ling et al., 2019). More recent studies have integrated EM with Wiener-process models involving measurement errors, unknown change points, and latent degradation states (Guan, Wei et al., 2022; P. Wang, Tang, Bae, & Xu, 2018). Beyond the standard EM framework, specialized implementations, such as embedded EM for simultaneous change-point and parameter estimation and EM-based hidden-state inference for multi-source fusion models, have further expanded its applicability to more complex multi-phase degradation problems (Han et al., 2026; Shao et al., 2023).

Besides MLE and EM-type methods, several estimation strategies have been developed for specific model structures. These include generalized least squares (GLS) combined with MLE for two-phase IG processes (Gu et al., 2026), MLE for estimating cross-phase covariance parameters in phase-dependent Wiener processes (B. Zhuang et al., 2026), and recursive nonlinear least squares, residual-sum-of-squares minimization, or nonlinear gradient optimization for models involving physical constraints or nonlinear parameterized degradation functions (C. Wang et al., 2025; J. Wang, Ma, & Zhang, 2025; Zhou et al., 2025). For multivariate degradation models with dependent performance indicators, Bayesian stepwise estimation and copula parameter estimation have also been adopted to jointly characterize degradation evolution and dependence structures (L. Fan et al., 2024).

Overall, likelihood-based estimation provides a well-established framework for parameter estimation, model comparison, and change-point identification. Parameter uncertainty

can be assessed using information-based, profile-likelihood, or bootstrap methods, although inference may become more challenging for models involving latent change points or random effects.

4.1.2. Bayesian inference

Bayesian inference treats unknown parameters as random variables and combines prior information with observed data to obtain posterior distributions (Gelman et al., 1995). Let $\mathcal{Y}$ denote the observed degradation data and $\boldsymbol{\theta}$ the parameter vector. The posterior distribution is given by

$$p(\boldsymbol{\theta} \mid \mathcal{Y}) \propto p(\mathcal{Y} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}),$$

where $p(\mathcal{Y} \mid \boldsymbol{\theta})$, $p(\boldsymbol{\theta})$, and $p(\boldsymbol{\theta} \mid \mathcal{Y})$ denote the likelihood, prior, and posterior distributions, respectively.

Existing studies have widely adopted Bayesian inference for multi-phase degradation models. Hierarchical Bayesian inference has been used to represent unit heterogeneity and random change points in Bayesian general path regression models (Bae, Yuan, Ning et al., 2015), Wiener-process models with measurement errors (P. Wang, Tang, Bae, & He, 2018; P. Wang, Tang, Bae, & Xu, 2018), and reparameterized or nonlinear IG processes with random effects (Z. Z. Yan et al., 2025; L. Zhuang et al., 2024). In Bayesian inference, posterior distributions are typically obtained through sampling-based algorithms, with Markov chain Monte Carlo (MCMC) and Gibbs sampling being the most commonly adopted approaches. When posterior computation becomes computationally expensive or analytically intractable, approximate Bayesian methods provide efficient alternatives. Representative examples include variational Bayes for multi-phase nonlinear Wiener processes (W. Lin et al., 2024) and adaptive Bayesian synthetic likelihood for two-phase TED processes with intractable likelihoods (Tian, Zhang et al., 2025).

Compared with MLE, Bayesian inference offers a natural framework for jointly quantifying uncertainty in model parameters and change-point locations and incorporating prior information, which can be particularly useful for limited degradation data or complex hierarchical models. However, Bayesian inference generally requires greater computational effort, especially when posterior sampling is needed, whereas MLE can be computationally more direct when the likelihood is tractable.

4.1.3. Recursive estimation

Recursive estimation sequentially updates degradation states or model parameters as new monitoring data become available, making it particularly suitable for online reliability assessment and RUL prediction in multi-phase degradation processes. Unlike batch estimation methods, recursive estimation updates model estimates as new observations arrive, typically avoiding repeated fitting to the full dataset. Let $\hat{\boldsymbol{\theta}}_t$ denote the estimate at time t and y_t the newly acquired observation. A general recursive estimation framework can be expressed as

$$\hat{\boldsymbol{\theta}}_t = \mathcal{F}\left(\hat{\boldsymbol{\theta}}_{t-1}, y_t\right),$$

where $\mathcal{F}(\cdot)$ denotes the recursive updating rule determined by the underlying estimation algorithm.

Existing studies mainly follow two recursive estimation routes. The first is Bayesian updating, in which posterior distributions are sequentially updated as new observations become

available. Representative studies include empirical Bayes updating for two-phase degradation regression (Chen & Tsui, 2013) and Bayesian updating that combines accelerated degradation testing with field data to update model parameters and change-point distributions (J. Wang, Wu et al., 2024). The second route relies on state-space models and recursive filtering, including the Kalman filter (KF), extended Kalman filter (EKF), unscented Kalman filter (UKF), and particle filter (PF). Representative developments include Kalman smoothing with parameter updating for two-phase Wiener processes (Guan, Wei et al., 2022), state-space models combined with change-point detection using EKF or UKF for online phase identification and RUL prediction (J. Wang, Ma, & Zhang, 2025; S. Wang, Zhang et al., 2026), and incremental or phase-aware Kalman filtering for phase transitions and changing noise structures (J. Li, Fan, Wang, Qiu et al., 2025; J. Li, Fan, Wang, Zhang et al., 2024).

Overall, recursive estimation enables degradation states and model parameters to be updated continuously as new observations arrive. Its effectiveness depends on reliable phase identification and suitable state-space or filtering formulations. In complex degradation environments, adaptive updating and model correction are often needed to reduce error accumulation over time.

4.2. Change-point detection

The identification of phase boundaries is a central task in multi-phase degradation analysis. Existing methods can be broadly divided into two categories according to how the phase structure enters the inferential procedure. The first explicitly searches or compares candidate change-point locations or segmentations using model-selection criteria or statistical tests, whereas the second treats change points or latent phase states as unknown quantities within a unified likelihood, posterior, or state-space formulation. The following subsections review these two classes of methods.

4.2.1. Search-based detection

Search-based methods are among the most widely used approaches for change-point identification in multi-phase degradation models. Their basic idea is to explicitly evaluate candidate change-point locations or segmentations and determine the optimal phase partition according to model-selection criteria, statistical tests, or related detection statistics. Information criteria, likelihood-ratio (LR) statistics, and residual-based measures are among the most commonly used tools. Model parameters may be re-estimated for each candidate change-point location during the search procedure.

Common information criteria include the Akaike information criterion (AIC) and the Bayesian/Schwarz information criterion (BIC/SIC). A general information criterion for a candidate change-point location τ can be expressed as

$$\mathrm{IC}(\tau) = -2\ell(\hat{\boldsymbol{\theta}}_\tau) + \mathrm{penalty}(\tau),$$

where $\ell(\hat{\boldsymbol{\theta}}_\tau)$ is the maximized log-likelihood of the piecewise model given τ, and the penalty term depends on the specific criterion adopted (e.g., AIC, or BIC/SIC). The estimated change-point location is selected as the candidate location that minimizes the information criterion. SIC has been widely adopted to identify change-point locations in two-phase IG and Wiener process models (Duan & Wang, 2017; Gu et al., 2026; Liang et al., 2024; J. Lin, Shen et al., 2025). To improve robustness, Qiao et al. (2025) proposed a modified information

criterion (MIC) for IG processes, while MIC/SIC has also been applied to failure-intensity processes and failure-mechanism switching (Bae, Mun et al., 2013; J. Li, Tian et al., 2020).

Besides information criteria, LR-based methods provide another search-based strategy for change-point detection (Lim & Bae, 2026). These methods compare the no-change model with a change-point model over candidate change-point locations. For a candidate location τ, let ℓ_0 denote the maximized log-likelihood under the no-change model and $\ell_1(\tau)$ the maximized log-likelihood under the model with a change point at τ. The LR statistic can then be written as

$$\mathrm{LR}(\tau) = 2\big[\ell_1(\tau) - \ell_0\big].$$

When the change-point location is unknown, the test is commonly based on the maximum of $\mathrm{LR}(\tau)$ over the candidate set, and the estimated change point is the corresponding maximizer. In two-phase Wiener degradation modelling, log-likelihood differences have been compared with an empirically determined threshold to select between one-phase and two-phase models (Prakash & Kaushik, 2020). Empirical-likelihood formulations similarly maximize an empirical LR statistic over candidate locations; when asymptotic critical values are unreliable for small samples, bootstrap resampling can be used to obtain empirical critical values (Cai et al., 2019).

Residual-based search provides another approach for change-point localization. For each candidate location, separate piecewise degradation functions are fitted to the observations before and after the candidate point, and the candidate minimizing the total residual sum of squares (RSS) is selected as the estimated change point (Yao & Zhu, 2026). Sequential statistics have also been used to detect changes in degradation increments. Representative examples include an adjusted CUSUM statistic combined with binary search for multiple-change-point identification in IG processes (M. Li et al., 2025), and filtering-derivative statistics evaluated over multiple window sizes through a multiple filter algorithm for change-point detection in two-phase TED processes (Tian, Zhang et al., 2025).

4.2.2. Joint-inference-based identification

Depending on the model structure, joint inference can be performed through likelihood-based estimation, Bayesian posterior inference, or latent-state filtering and updating. This strategy is particularly useful when phase-specific parameters and change-point locations are statistically coupled or when change-point uncertainty needs to be retained for subsequent reliability analysis and RUL prediction. Existing studies have applied this idea to general path models and stochastic-process models. In two-phase settings, change points have been incorporated into Bayesian general path regression models (Bae, Yuan, Ning et al., 2015), condition-monitoring and mixed-effects regression models (Chen & Tsui, 2013; Son et al., 2015), and Wiener or gamma process models (Ling et al., 2019; P. Wang, Tang, Bae, & He, 2018). Related formulations have also been developed for two-phase IG processes and Gaussian process models (S. Chen et al., 2025; Z. Chen et al., 2021; L. Zhuang et al., 2024). For multiple-change-point settings, joint inference becomes more challenging because both change-point structures and the current segment state need to be inferred. Exact Bayesian or recursive posterior updating frameworks have been developed to handle such settings in regression-based and Wiener-process degradation models (Y. Wen, Wu, Das et al., 2018; Y. Wen, Wu, Zhou et al., 2019).

Overall, search-based methods are relatively straightforward when the number of candidate change points is limited, but repeated model fitting can become computationally

demanding for more complex phase structures. In contrast, joint-inference methods can account for change-point uncertainty within a unified inferential framework, although they may involve greater computational complexity and stronger dependence on model specification. The choice between these approaches therefore depends on the phase structure, computational requirements, and the importance of uncertainty quantification for subsequent reliability and RUL analysis.

5. RUL prediction and decision-making

Building on the phase structures identified and model parameters estimated using the methods reviewed in Section 4, multi-phase degradation models can be used for RUL prediction and maintenance decision-making. This section reviews how phase-specific degradation information is incorporated into these applications.

5.1. RUL prediction methods

Remaining useful life (RUL) prediction aims to estimate the time remaining before failure using current monitoring information. Let T_f denote the failure time. The RUL at time t is defined as $\mathrm{RUL}(t) = T_f - t,\ t < T_f$. If failure is defined as the first time that the degradation state reaches or exceeds a threshold D, the failure time can be formulated as the first hitting time (FHT), $T_f = \inf\{s \geq 0 : X(s) \geq D\}$, where $X(s)$ denotes the degradation state. For performance-based degradation indicators, failure can similarly be defined as the first time at which the performance falls below a prescribed threshold. Based on this definition, existing RUL prediction methods can be broadly classified into model-based probabilistic methods and supervised-learning-based methods.

(1) *Model-based probabilistic methods.* These methods establish a multi-phase degradation model and infer the conditional RUL distribution and associated quantities, such as its expectation and prediction intervals, based on the FHT distribution (Wu et al., 2026; A. Xu et al., 2026). Existing studies can be broadly divided into analytical RUL inference and approximation-based RUL inference.

Analytical RUL inference derives explicit formulas or analytical integral representations for FHT or RUL distributions based on multi-phase stochastic processes. For Wiener processes with constant drift and diffusion coefficients, the FHT to a fixed failure boundary follows an IG distribution when the drift is directed toward that boundary. This property provides a basis for analytical RUL inference in two-phase Wiener-process models, although phase transitions must be accounted for when deriving the overall FHT distribution. Closed-form failure-time and RUL distributions have been derived for change-point Wiener processes with random effects or measurement errors (P. Wang, Tang, Bae, & He, 2018; P. Wang, Tang, Bae, & Xu, 2018), whereas analytical integral representations have been obtained when random jumps at change points are considered (H. Gao et al., 2020). For IG-based models, reparameterized, nonlinear, and random-volatility formulations have been developed for two-phase degradation processes, where RUL distributions are commonly expressed through analytical integral or convolution representations rather than closed forms (Liang et al., 2026; Z. Z. Yan et al., 2025; L. Zhuang et al., 2024).

Approximation-based RUL inference provides practical alternatives when analytical FHT or RUL distributions are unavailable or computationally difficult to evaluate. Representative approaches include analytical approximations and simulation-based methods. For example,

H. Zhang, Zhou et al. (2019) approximated the RUL distribution for fractional Brownian-motion degradation using weak convergence and time-space transformations, Tian, Zhang et al. (2025) employed saddlepoint and Birnbaum-Saunders approximations for lifetime inference under a two-phase TED process, and Duan et al. (2021) estimated the RUL distribution for multi-mode degradation driven by fractional Lévy stable motion using Monte Carlo simulation.

Overall, model-based probabilistic methods provide physically interpretable RUL distributions together with uncertainty quantification. However, their performance depends on the validity of degradation-model assumptions, while analytical derivation or numerical evaluation becomes increasingly challenging for complex phase-transition mechanisms or stochastic dynamics.

(2) *Supervised-learning-based methods.* Unlike model-based probabilistic methods, supervised learning methods formulate RUL prediction as a regression or sequence-learning problem without explicitly deriving FHT distributions. Let $\mathbf{x}_{1:t}$ denote the monitoring data collected up to time t. A general prediction model can be written as $\widehat{\mathrm{RUL}}(t) = f(\mathbf{x}_{1:t})$, where $\mathbf{x}_{1:t}$ may include raw sensor signals, degradation features, health indicators (HIs), degradation-stage information, or multi-source features. These methods generally require run-to-failure trajectories or labelled RUL targets for training and are suitable for complex degradation processes whose physical mechanisms are difficult to model explicitly. Existing studies incorporate degradation-stage information into RUL prediction mainly through two complementary strategies: stage-informed target construction and stage-integrated prediction models.

Stage-informed target construction incorporates degradation-stage information into the training labels or intermediate health representations used for RUL prediction. Instead of relying only on failure-time-based labels, these methods use HIs, health-state representations, stage labels, or individualized change-point information to construct degradation-aware supervision before network training (Arunan et al., 2024; Rath et al., 2025; Shi & Chehade, 2021; J. Yan et al., 2022). For example, Shi and Chehade (2021) first detected unit-specific change points using a long short-term memory (LSTM) classifier and then used them to construct piecewise HIs for subsequent LSTM-based prediction. Such designs allow the supervision signal to reflect heterogeneous phase-transition times across units rather than assuming a common transition point. Stage-integrated prediction models incorporate degradation-stage information into the prediction architecture or procedure itself. Instead of using stage information only to construct supervision signals, these methods use stage classifications, state-transition information, or stage-specific features to guide feature learning and RUL estimation. Representative studies include health-stage-specific prediction frameworks (Ensarioğlu, 2025), Transformer-based stage recognition followed by stage-dependent RUL prediction (J. Zheng et al., 2026), graph neural network models that use stage-classification results to construct prediction graphs (Huang et al., 2025), and integrated architectures combining state-change detection with subsequent RUL prediction (Z. Gao et al., 2025; Gupta et al., 2023). Compared with stage-informed target construction, these approaches allow stage information to affect the prediction mechanism more directly, but their performance may also become more sensitive to stage-identification errors.

Overall, supervised-learning-based methods can capture complex degradation patterns from high-dimensional sensor data, but several challenges remain. First, labelled run-to-failure data are often scarce, particularly for highly reliable systems. Second, domain shifts

Table 3. Comparison of representative stage-aware data-driven RUL prediction methods.

Study	Prediction framework	Use of stage information	Evaluation metrics
Shi and Chehade (2021)	Dual-LSTM	Unit-specific CP detection and post-CP HI construction for RUL prediction	SF, RMSE, relative prediction error
Rath et al. (2025)	Logistic regression and BiLSTM	Feature-based CP detection guides HI construction and subsequent RUL prediction	RMSE, MAE, percentage error
J. Yan et al. (2022)	Attention-LSTM and HI-based similarity estimation	CP identification guides HI construction, forecasting, and RUL distribution estimation	Mon, RMSE, PSF
Ensarioğlu (2025)	Health-stage classifier; CNN-LSTM and LSTM	Stage classification selects separate RUL predictors for healthy and degraded states	RMSE, Score
J. Zheng et al. (2026)	Transformer and TBiGAN	Adaptive stage identification guides subsequent RUL prediction	MAE, RMSE, R^2, Score
Huang et al. (2025)	GCN-LSTM and AAGL	Stage classification guides graph construction for RUL prediction	RMSE, MAE, MAPE, R^2, adjusted R^2
Z. Gao et al. (2025)	CTN and VTN	State-change detection identifies degradation-stage data for RUL prediction	MAE, Score, NRMSE

Note: TBiGAN: TCN-BiGRU-global-attention network; AAGL: adaptive attention GraphSAGE-LSTM; CTN: contrastive Transformer network; VTN: versatile Transformer network; SF: scoring function; PSF: penalty score function; Mon: monotonicity; NRMSE: normalized root mean square error.

across operating conditions may limit model generalization. Third, limited physical interpretability can make it difficult to relate learned features to underlying degradation mechanisms. Table 3 compares representative stage-aware methods in terms of their prediction frameworks, use of degradation-stage information, and evaluation metrics.

5.2. Maintenance and operational decision-making

While multi-phase degradation models characterize degradation evolution and support RUL prediction, their practical value is ultimately realized through decision-making. By identifying degradation phases and transition behaviour, these models provide phase-specific information, such as the current degradation stage, phase duration, phase-dependent risk, and updated lifetime distribution, for maintenance optimization. Compared with single-phase models, decision policies based on multi-phase degradation can account for changes in degradation rate, failure risk, and intervention value across different stages. Existing studies can be broadly grouped into replacement optimization based on degradation prognosis, joint optimization of inspection and maintenance, and extended operational decision-making.

(1) *Prognosis-driven replacement optimization.* These methods use prognostic information derived from multi-phase degradation models, including reliability functions, lifetime distributions, and RUL estimates, to optimize replacement timing, inspection policies, or control limits. Compared with conventional condition-based maintenance, these methods incorporate phase-specific prognostic information into replacement decisions, enabling maintenance actions to adapt to the evolving degradation process. Representative studies include an adaptive replacement policy driven by the posterior RUL distribution (L. Zhuang et al., 2024), while J. Wang, Tan et al. (2023) jointly optimized inspection intervals, control limits, and safety factors using real-time reliability updating together with expected RUL within a two-phase Wiener degradation framework. Related studies further extended control-limit replacement policies to change-point shock models by combining reliability analysis with long-run average cost optimization (Chadjiconstantinidis & Eryilmaz, 2023; Eryilmaz & Kan, 2019).

(2) *Inspection-maintenance joint optimization*. These methods jointly optimize inspection scheduling and maintenance decisions under phase-dependent degradation models. Because degradation rates and failure risks differ across phases, the preferred inspection frequency and maintenance action also change with the system state. Representative studies have therefore developed joint inspection–maintenance policies with different decision variables. For example, J. Wang, Ma et al. (2024) jointly optimized the inspection interval, degradation threshold, and age limit for a two-phase continuously degrading system subject to time- and state-dependent false-positive and false-negative inspection errors. Song et al. (2023) proposed a two-phase inspection and preventive-maintenance policy under a Wiener degradation process by optimizing inspection intervals and preventive-replacement thresholds. L. Yang et al. (2018) integrated a general path degradation model with an NHPP shock process and jointly optimized the replacement age, degradation control limit, and state-dependent inspection intervals for a normal-defective-failed system subject to competing soft and hard failures.

(3) *Phase-aware operational decision-making*. Beyond maintenance scheduling, multi-phase degradation information has also been incorporated into broader operational decision-making by utilizing phase-specific degradation and system-state information. Representative studies include a multi-criteria mission-abort policy that combines degradation control limits and defect-phase duration to balance mission reliability and system survivability under a two-phase Wiener degradation framework (Zhao, Fan et al., 2021). For complex systems, Lee and Pan (2017) developed a predictive maintenance framework that combines Markov-chain degradation models with Bayesian-network reliability prediction, while Sun and Vatn (2024) optimized condition-based inspection and maintenance strategies using a phase-type maintenance model incorporating deterministic maintenance delays. These studies demonstrate that multi-phase degradation models can support operational decisions beyond maintenance scheduling by incorporating phase-specific degradation and state information into decision policies.

Overall, maintenance and operational decision-making represents the downstream stage of multi-phase degradation analysis, where phase information and prognostic results are translated into replacement, inspection, and operational policies. Previous studies show a clear progression from prognosis-assisted maintenance optimization toward phase-aware decision-making that explicitly incorporates degradation-stage characteristics into operational decisions.

6. Conclusions and future research directions

Multi-phase degradation analysis provides a systematic framework for systems whose degradation behaviour changes during operation. Its main value lies not merely in introducing change points, but in representing degradation as a sequence of phases with distinct evolution rates, stochastic characteristics, failure mechanisms, and risk levels, thereby relaxing the homogeneous-evolution assumption of conventional single-phase models. Existing research has developed from general path models to stochastic-process formulations based on Wiener, gamma, IG, Gaussian process, and shock-based models, together with advances in likelihood-based estimation, Bayesian inference, recursive updating, and change-point detection. These developments have further supported probabilistic and learning-based RUL prediction, as well as replacement, inspection, maintenance, and operational decision-making that explicitly use phase-specific degradation information.

Despite substantial progress, several important challenges remain in multi-phase degradation analysis. Future research may proceed along the following directions.

(1) *Mechanism-informed phase-aware degradation modelling.* Some studies have incorporated engineering mechanisms or physical degradation laws into multi-phase degradation models, including bi-exponential degradation paths, degradation propagation models, and Paris-law-based state-space models (H. Wang, Liao et al., 2022; J. Wang, Ma, & Zhang, 2025; Yuan et al., 2016). However, even in these studies, phase transitions are generally identified through statistical change-point detection, monitoring-signal variations, or empirical rules, while their links to underlying degradation mechanisms remain insufficiently established. Future research should establish more explicit links between physical degradation mechanisms and statistical phase transitions by integrating physical indicators, stress responses, and experimental observations into degradation modelling. For example, Paris' law for fatigue-crack propagation, Arrhenius-type relations for temperature-accelerated degradation, and electrochemical aging relations for batteries could be incorporated into phase-specific degradation models. A key challenge is to identify physically interpretable criteria for phase transitions based on changes in degradation mechanisms or operating regimes, rather than relying solely on statistical change-point detection.

(2) *Phase-aware uncertainty quantification.* Existing studies have considered uncertainty in change-point locations, model parameters, degradation states, and RUL prediction (Brumm et al., 2025; J. Wang, Ma, & Zhang, 2025; J. Wang, Wu et al., 2024; Y. Wen, Wu, Zhou et al., 2019). However, these sources are often analyzed separately, and their combined effects on subsequent prediction and decision-making remain insufficiently studied. Particular attention should be given to phase-transition uncertainty, including uncertainty in transition timing, phase membership, and future phase evolution. Unlike ordinary parameter uncertainty, phase-transition uncertainty may alter the active degradation model itself, thereby affecting future degradation trajectories and failure-time distributions. Future research could develop unified probabilistic frameworks, such as hierarchical Bayesian models for jointly inferring phase-specific parameters and change points, and switching state-space models for representing uncertain phase membership and transitions. Posterior predictive simulation, potentially combined with sequential Monte Carlo for online updating, could further propagate these uncertainties into RUL distributions and maintenance-related risk measures.

(3) *Generalizable phase-aware prediction.* Existing studies have improved phase-aware prediction by incorporating richer monitoring information and cross-condition learning strategies. Representative developments include multi-feature HI construction, CNN-assisted multi-stage degradation modelling, multi-source signal fusion, and domain adaptation for cross-condition prediction (Z. Fan et al., 2025; Guan, Zuo et al., 2024; S. Sun et al., 2026; Xiao et al., 2025; X. Zhang et al., 2023). However, existing methods mainly address specific operating conditions or individual transfer tasks, while phase-aware prediction across heterogeneous data sources, operating conditions, and units remains challenging. Future research could explore domain-adversarial learning to extract transferable degradation features and meta-learning to facilitate adaptation when target-domain observations are limited. More importantly, phase-aware transfer should account for differences in transition timing, phase-specific degradation rates, and the number or characteristics of degradation phases, rather than assuming direct phase correspondence across domains.

(4) *AI-enabled phase-aware degradation analysis.* Artificial intelligence (AI) techniques have been increasingly incorporated into multi-phase degradation analysis for change-point detection, feature learning, degradation-state representation, and RUL prediction (Arunan

et al., 2024; Shi & Chehade, 2021; P. Yang et al., 2026). However, existing studies primarily focus on individual identification or prediction tasks, while the integration of physical degradation mechanisms, uncertainty quantification, and maintenance decision-making remains limited. Future research could explore Transformer architectures with phase-aware attention to capture stage-dependent temporal patterns, graph attention networks to model interactions among multiple sensors or degradation indicators, and physics-informed neural networks to incorporate phase-specific degradation laws or physical constraints. Moreover, multi-task architectures with shared encoders and task-specific prediction heads could jointly perform phase identification, degradation-state estimation, and RUL prediction, providing an integrated basis for subsequent maintenance decision-making.

Author contributions

CRediT: **Liangliang Zhuang:** Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft; **Nan Liu:** Data curation, Resources, Validation; **Di Zhu:** Data curation, Validation; **Ancha Xu:** Conceptualization, Funding acquisition, Writing – review & editing

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The research was supported by the National Natural Science Foundation of China [grant nos. 72601209, 72571246, 12531013], the Zhejiang Provincial Natural Science Foundation of China [grant no. LZ24A010002], the China Postdoctoral Science Foundation [grant no. 2026M794714], the Postdoctoral Fellowship Program of CPSF [grant number GZC20260433], the Summit Advancement Disciplines of Zhejiang Province (Zhejiang Gongshang University—Statistics), and the Collaborative Innovation Center of Statistical Data Engineering Technology & Application.

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